12-16-13 Math 65

TEST 5 REVIEW

via Q12, Q13, Q14

Quiz 12 problem

$x = x^2 - 30$

30 less

$x = x^2 - 30$

$-x \quad -x$

$0 = x^2 - x - 30 = x^2 + (-1)\cdot x - 30$

$0 = (x + A)(x - B)$

$A - B = -1$

$A \cdot B = 30$

$A = 5$, $B = 6$) $5 - 6 = -1$, $5 \cdot 6 = 30$

NOTE: $5 \cdot (-6) = -30$

$$0 = (x+5)(x-6)$$

$$x+5=0 \text{ or } x-6=0$$

$$x=-5 \text{ or } x=6$$

practice:

(i) a number is 12 less than its square.

$$x = x^2 - 12$$

$$0 = x^2 - x - 12$$

$$0 = (x+3)(x-4)$$

$$x=-3 \text{ or } x=4$$

(ii) a number is 42 less than its square

$x = x^2 - 42$

$0 = x^2 - x - 42$

$0 = (x+6)(x-7)$

$x = -6$ or $x = 7$

Problem 1

(a.) LCM of $3x^4$ and $9x^2$

F	G
3	2
x	4

$3 = 3$

$9 = 3 \cdot 3$

$3x^4 = 3 \cdot x^4$

$9x^2 = 3 \cdot 3x^2$

$3^2 \cdot x^4$

$= 9x^4 = LCM$

(b.) $15x^4y^7$, $25x^3y^9$

F	G
3	1
5	2

$15 = 3 \cdot 5$

$25 = 5 \cdot 5$

$LCM = 3^1 \cdot 5^2 \cdot x^4 y^9$

$\Rightarrow LCM = \boxed{75} \cdot x^4 y^9$

$= 75x^4y^9$

(2) $\frac{6}{x} + \frac{2}{x^2}$

$LCD = LCM = x^2$

F	6
x	2

$\Rightarrow LCM = x^2$

$\frac{6}{x} \cdot \frac{\boxed{x}}{\boxed{x}} + \frac{2}{x^2} \cdot \frac{1}{1}$

$\frac{x^2}{x} = \boxed{x}$; $\frac{x^2}{x^2} = 1$

$\frac{6x}{x^2} + \frac{2}{x^2} = \frac{6x+2}{x^2}$

(c) $\frac{1}{3x^4} + \frac{1}{9x^2}$ (3) $\Rightarrow$ LCD $= 9x^4$ (2)

see (a)

$$\frac{9x^4}{3x^4} = 3, \quad \frac{9x^4}{9x^2} = x^2$$

$$\frac{1}{3x^4} \cdot \frac{3}{3} + \frac{1}{9x^2} \cdot \frac{x^2}{x^2}$$

$$= \frac{3}{9x^4} + \frac{x^2}{9x^4}$$

$$= \frac{3 + x^2}{9x^4}$$

(6) $\frac{1}{15x^4y^7} + \frac{1}{25x^3y^9}$

LCD $= \overbrace{75x^4y^9}^{(6)}$

$\frac{75x^4y^9}{15x^4y^7} = 5y^2$

$\frac{75x^4y^9}{25x^3y^9} = 3x$

$\frac{1}{15x^4y^7} \cdot \frac{5y^2}{5y^2} + \frac{1}{25x^3y^9} \cdot \frac{3x}{3x}$

$= \boxed{\frac{5y^2 + 3x}{75x^4y^9}}$

6.5

(4) $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-6} = \dfrac{\frac{1}{x}+\frac{4}{1}}{\frac{1}{x}-\frac{6}{1}}$ } LCD of x, 1, x and 1 LCD = x.

$\downarrow$

$= \dfrac{x\cdot\left(\frac{1}{x}+\frac{4}{1}\right)}{x\cdot\left(\frac{1}{x}-6\right)} = \dfrac{\frac{x}{x}+4\cdot x}{\frac{x}{x}-6x} = \boxed{\dfrac{1+4x}{1-6x}}$

SIMPLIFIED

(5.)

$$\frac{\frac{5}{x}-\frac{3}{x^2}}{\frac{4}{x^2}}$$; LCD $= x^2$

$$\frac{x^2\cdot\left(\frac{5}{x}-\frac{3}{x^2}\right)}{x^2\cdot\left(\frac{4}{x^2}\right)} = \boxed{\frac{5x-3}{4}}$$

SIMPLIFIED

(6) $\dfrac{7+\frac{1}{4}}{1+\frac{1}{2}}$; Math 104

$= \dfrac{\frac{7}{1}+\frac{1}{4}}{\frac{1}{1}+\frac{1}{2}}$ LCD of 1, 4, 1, and 2 LCD = 4

$\Rightarrow \dfrac{4\cdot\left(\frac{7}{1}+\frac{1}{4}\right)}{4\cdot\left(\frac{1}{1}+\frac{1}{2}\right)} = \dfrac{28+1}{4+2} = \dfrac{29}{6}$

(7a)

$$\frac{\frac{1}{x}+3}{\frac{1}{x}-7} = \frac{\frac{1}{x}+\frac{3}{1}}{\frac{1}{x}-\frac{7}{1}}$$

$\cdot x = X$

$$\rightarrow \frac{x\cdot\left(\frac{1}{x}+\frac{3}{1}\right)}{x\cdot\left(\frac{1}{x}-\frac{7}{1}\right)}$$

$\frac{x}{x}=1$

$x\cdot\frac{7}{1}=7x$

$\frac{x}{x}=1$

$x\cdot\frac{3}{1}=3x$

$$\Rightarrow = \boxed{\frac{1+3x}{1-7x}}$$

(12

(8)

$$\frac{\frac{5}{x} - \frac{2}{x^2}}{\frac{6}{x}} \cdot x^2 = x^2$$

$$\frac{x^2 \cdot \left(\frac{5}{x} - \frac{2}{x^2}\right)}{x^2 \cdot \left(\frac{6}{x}\right)}$$

$x^2 \cdot \frac{5}{x} = 5x$

$x^2 \cdot \frac{2}{x^2} = 2$

$x^2 \cdot \frac{6}{x} = 6x$

$$\frac{5x - 2}{6x}$$

9.

$$\frac{3+\frac{1}{4}}{1+\frac{1}{2}} = \frac{\frac{3}{1}+\frac{1}{4}}{\frac{1}{1}+\frac{1}{2}}$$

LCD = 4

$4 \cdot \frac{3}{1} = 12$

$4 \cdot \frac{1}{4} = 1$

$4 \cdot \frac{1}{1} = 4$

$4 \cdot \frac{1}{2} = 2$

$$\frac{4\left(\frac{3}{1}+\frac{1}{4}\right)}{4\left(\frac{1}{1}+\frac{1}{2}\right)} = \frac{12+1}{4+2} = \frac{13}{6}$$

Problems 10, 11, 12 are the same

(10.) $x + \frac{5}{x} = -6$

$\Rightarrow \frac{x}{1} + \frac{5}{x} = \frac{-6}{1}$

LCD of 1, x and 1 $= x$

$x \cdot \left(\frac{x}{1} + \frac{5}{x}\right) = x \cdot \left(\frac{-6}{1}\right)$

$x \cdot \frac{x}{1} + x \cdot \frac{5}{x} = -6x$

$x^2 + 5 = -6x$

$+6x \qquad +6x$

$x^2 + 6x + 5 = 0$

(15)

$$x^2+6x+5=0$$

$$(x+1)\cdot(x+5)=0$$

$1 \cdot 5 = 5$

$x+1=0$ OR $x+5=0$

$x=-1$ OR $x=-5$

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(16) $x+\frac{3}{x}=-4$

LCD $=x$

$$\frac{x}{1}+\frac{3}{x}=\frac{-4}{1}$$

$$x\cdot\left(\frac{x}{1}+\frac{3}{x}\right)=x\cdot\left(\frac{-4}{1}\right)$$

(16

11) 

$$x \cdot \left(\frac{x}{1} + \frac{3}{x}\right) = x \cdot \left(-\frac{4}{1}\right)$$

$$x^2 + 3 = -4x$$

F L

$$x^2 + 3 = -4x$$

$$+4x \quad +4x$$

$$x^2 + 4x + 3 = 0$$

$$(x+1)\cdot(x+3) = 0$$

$$x+1=0 \text{ OR } x+3=0$$

$$x = -1 \text{ OR } x = -3$$

(12.) $x + \frac{6}{x} = -7$

(17

$$\frac{x}{1} + \frac{6}{x} = -\frac{7}{1}$$

$$x \cdot \left(\frac{x}{1} + \frac{6}{x}\right) = x \cdot \left(-\frac{7}{1}\right)$$

$$x^2 + 6 = -7x$$

$$+7x \quad +7x$$

$$x^2 + 7x + 6 = 0$$

$$(x+1)\cdot(x+6) = 0$$

$x+1=0$ or $x+6=0$

$x=-1$ or $x=-6$

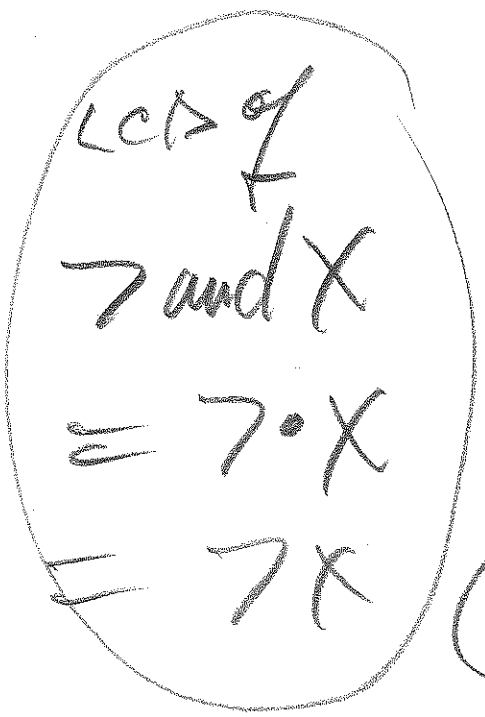
Quiz 13

(15.) $\frac{x}{7} - \frac{7}{x} = 0$

$7x \cdot \left(\frac{x}{7} - \frac{7}{x}\right) = 7x \cdot 0$

$7x \cdot \frac{x}{7} - 7x \cdot \frac{7}{x} = 0$

$x^2 - 49 = 0$



Quiz B

(5.)

$$x^2 - 49 = 0$$

$$x^2 - 7^2 = 0$$

$$(x+7)\cdot(x-7) = 0$$

$$x+7 = 0 \text{ OR } x-7 = 0$$

$$x = -7 \text{ OR } x = 7$$

$$x = \pm 7$$

(20

(16)

$$(\text{rate}) \cdot t = 1$$

ADD rates of 2 workers

$$\left(\frac{1}{8} + \frac{1}{6}\right) \cdot t = 1$$

↑ working together

LCD = 24

$$\frac{t}{8} + \frac{t}{6} = \frac{1}{1}$$

$$24 \cdot \left(\frac{t}{8} + \frac{t}{6}\right) = 24 \cdot \frac{1}{1}$$

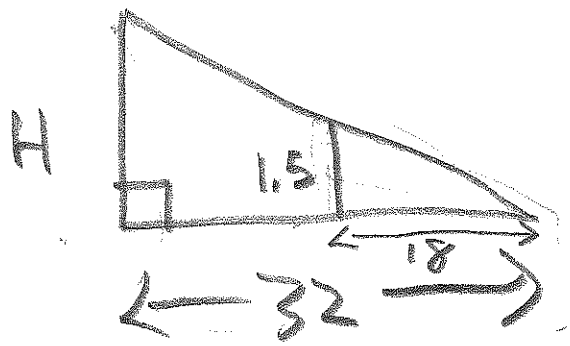
$$24\cdot\frac{t}{8}+24\cdot\frac{t}{6}=24$$ (2)

$$3t+4t=24$$

$$7t=24$$

$$t=\frac{24}{7}\text{ hours}$$

(17.)



cross-multiply (10+)

$$\frac{H}{1.5}=\frac{32}{18}\Rightarrow 18H=48$$

$$H=\frac{48}{18}$$

(17.) $H = \frac{48}{18} = \frac{24}{9} = \frac{8}{3}$ (22)

$H = \frac{8}{3} = 3\frac{2}{3}$

$H = 3.666\ldots$

$H = 3.67$

$H = 3.7$

8.1

(18.) Both

(19.) square roots of 36, 100.

NUMBER	positive SQUARE ROOT	neg. SQ. ROOT
36	$6 \rightarrow 6^2 = 36$	$-6 \rightarrow (-6)^2 = 36$
100	$10 \rightarrow 10^2 = 100$	$-10 \rightarrow (-10)^2 = 100$

36 → square roots are 6, -6

" " " " 10, -10

100

Note: $\sqrt{36} = 6$ = pos. sq. root

$-\sqrt{36} = -6$ = neg. sq. root

$\sqrt{36}$ = pos. sq. root of 6

$-\sqrt{36}$ = neg. sq. root of 6

(20) $\sqrt{81}$ = POS. SQ. ROOT of 81

= SQ. ROOT of 81.

$\sqrt{81} = 9$ since $9^2 = 81$

practice: (i) $\sqrt{9} = 3 \rightarrow 3^2 = 9$

(ii) $\sqrt{25} = 5 \rightarrow 5^2 = 25$

(iii) $\sqrt{169} = 13 \rightarrow 13^2 = 169$

(iv) $\sqrt{144} = 12 \rightarrow 12^2 = 144$

(2w) $-\sqrt{25}$

$= -5$

NOTE: $-\sqrt{25}$

$= (-1) \cdot \sqrt{25}$

$= (-1) \cdot 5$

$= -5$

$$-\sqrt{25} = -5$$

(i) $-\sqrt{9} = -3$

(ii) $-\sqrt{64} = -8$

(iii) $-\sqrt{169} = -13$

(22) (a) $\sqrt{x^2} = x, \; x \geq 0$

$\sqrt{}$ undoes 2 exponent.

$\sqrt{x^2} = |x|$

(b) $\sqrt{9x^2} = \sqrt{3^2 \cdot x^2}$

$= \sqrt{(3x)^2} = 3x, \quad x \geq 0$

using $3^2x^2 = (3x)^2$

$\sqrt{}$ undoes (2) exponent

$= \sqrt{(3x)^2} = |3x|$

(c) $\sqrt{64x^2} = \sqrt{8^2x^2}$

$= \sqrt{(8x)^2}$

$= \boxed{8x}, \quad x \geq 0$

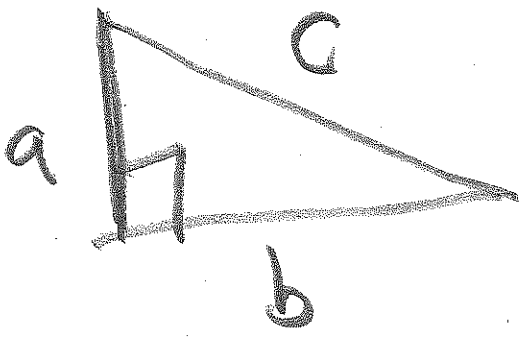
$\sqrt{}$ undoes (2)

(23.) $\sqrt{(x-5)^2} = (x-5)$, $x-5 \geq 0$

$\sqrt{}$ index (2)

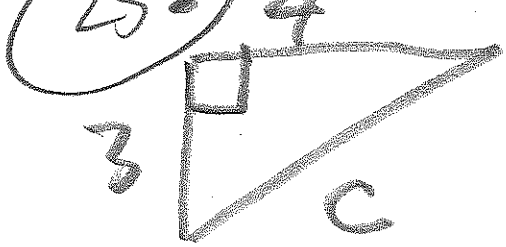
(24.) (b) ← choose b

$c^2 = a^2 + b^2$



← sec. 5.2

(25.)



$\Rightarrow c^2 = 3^2 + 4^2$

$c^2 = 9 + 16$

$c^2 = 25$

$$c^2 = 25$$

new element of thought:

Take SQUARE ROOT of BOTH SIDES

$$\sqrt{c^2} = \sqrt{25}$$

$c \geq 0$ *

$$c = 5 \quad \text{since } 5^2 = 25$$

$\sqrt{\ }$ undoes 2 exp.

$$\boxed{c = 5}$$

NO negative roots

$$\boxed{* \sqrt{c^2} = c, \; c \geq 0}$$

9.1

(ex) $x^2 = 16$

$x = -\sqrt{16}$ or $x = +\sqrt{16}$

$\rightarrow x = \pm\sqrt{16}$ incomplete

$\sqrt{16} = 4$ $(4^2 = 16)$

$$x = \pm 4$$

(27) $x^2 = 25$

$x = \pm\sqrt{25}$

$\boxed{x = \pm 5}$

since $\sqrt{25} = 5$

$(5^2 = 25)$

(28) $x^2 = 13$

$\boxed{x = \pm\sqrt{13}}$

$\boxed{x = -\sqrt{13} \text{ OR } x = \sqrt{13}}$

(29) $4x^2 = 100 \Rightarrow \frac{4x^2}{4} = \frac{100}{4}$

$x^2 = 25$

$x = \pm\sqrt{25}$

$x = \pm 5$

(30) $7x^2 = 21$

$x^2 = \frac{21}{7} = 3$

$x = \pm\sqrt{\frac{21}{7}} = \pm\sqrt{3}$

(30.)

$$7x^2 = 21$$

$$x^2 = \frac{21}{7}$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

(31.) (a) $(x-2)^2 = 36$

$$\boxed{X}^2 = K \Rightarrow \boxed{X} = \pm\sqrt{K}$$

$\boxed{X}$ = any expression

31 (a)

$$(x-2)^2 = 36$$

$$(x-2) = \pm\sqrt{36}$$

$$(x-2) = \pm 6$$

$$x-2 = \pm 6$$

$$+2 \qquad +2$$

$$x = 2 \pm 6$$

$$x = 2+6 \text{ OR } x = 2-6$$

$$x = 8 \text{ OR } x = -4$$

(b) $(x+6)^2 = 81$

$(x+6) = \pm\sqrt{81}$

$x+6 = \pm 9$

$-6 \quad -6$

$x = -6 \pm 9$

$x = -6-9$ or $x = -6+9$

$x = -15$ or $x = 3$

(31) (c)

$$(x-3)^2 = 7$$

$$(x-3) = \pm\sqrt{7}$$

$$x-3 = \pm\sqrt{7}$$

$$+3 \qquad +3$$

$$x = 3 \pm \sqrt{7}$$

(52.) $ax^2+bx+c=0$

CHOOSE (a.) → (a.) $x=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}$

(53.) $x^2+3x-40=0$

↓

$(1)\cdot x^2+(3)\cdot x+(-40)=0$

$ax^2+bx+c=0$

$a=1,\ b=3,\ c=-40$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2\cdot a}$

(33.)

$$\frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-40)}}{2 \cdot 1}$$

$$= \frac{-3 \pm \sqrt{9 + 160}}{2}$$

$$= \frac{-3 \pm \sqrt{169}}{2}$$

$$= \frac{-3 \pm 13}{2}$$

$$= \frac{-3+13}{2} \quad \text{OR} \quad \frac{-3-13}{2}$$

$$\frac{10}{2} \text{ OR } \frac{-16}{2} = \boxed{5 \text{ OR } -8}$$

Example practice:

$$x^2 + 6x - 1 = 0$$

$$a = 1,\ b = 6,\ c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 (-1)}}{2 \cdot 1}$$

$$x = \frac{-6 \pm \sqrt{36 + 4}}{2}$$

$$x = \frac{-6 \pm \sqrt{40}}{2}$$

Quiz 14

(40

(21) $\sqrt{3} \cdot \sqrt{13} = \sqrt{3 \cdot 13} = \sqrt{39}$

$$\sqrt{A} \cdot \sqrt{B} = \sqrt{A \cdot B}$$

(22) $\sqrt{17} \cdot \sqrt{17} = \sqrt{17 \cdot 17}$

$= \sqrt{17^2}$

$\sqrt{\text{undoes } 2 \text{ exp.}}$

$= 17$

note: $\sqrt{17^2} = 17$

$\sqrt{8^2} = 8$

$\sqrt{6.2^2} = 6.2$, ETC

(23.) $\sqrt{20} = \sqrt{2^2 \cdot 5}$

MINE for perfect squares.

$20 = 4 \cdot 5$

$\downarrow$

$= 2^2 \cdot 5$

$\sqrt{2^2 \cdot 5}$ use $\sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$

$\sqrt{2^2 \cdot 5} = \sqrt{2^2} \cdot \sqrt{5}$

$\downarrow$

$2 \cdot \sqrt{5}$

(24.) $\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6}$

$\downarrow$

$2 \cdot \sqrt{6}$

(25.) $\sqrt{36x^2} = \sqrt{6^2 x^2}$

use: $\sqrt{36} = 6$

use: $6^2 x^2 = (6x)^2$

$= \sqrt{(6x)^2}$

$= 6x$

$\sqrt{}$ undoes $\geq$ exp.

(26) $\sqrt{t^{60}} = \sqrt{(t^{30})^2}$

$t^{60} = (t^{30})^2$

$\sqrt{\quad}$ undoes (2)

$\sqrt{(t^{30})^2} = \boxed{t^{30}}$

math 55 TRICK: $\sqrt{t^{60}} = t^{\frac{60}{2}} = t^{30}$

(27) $\sqrt{6} \cdot \sqrt{3}$

$= \sqrt{6 \cdot 3}$

$= \sqrt{18}$ good BUT incomplete.

"mine" for perfect square.

$18 = 2 \cdot 9 = 2 \cdot 3^2$

$\sqrt{18} = \sqrt{2 \cdot 3^2} = \sqrt{2} \cdot \sqrt{3^2}$

$= \sqrt{2} \cdot 3$

$= 3\sqrt{2}$