

8-30-13

x and t
↓
MOLECULE VIBRATES

(1)

CH16

wave:

$$y(x,t) = A \cos(\omega t - kx)$$

= instantaneous displacement

y (from

EQUILIBRIUM) in

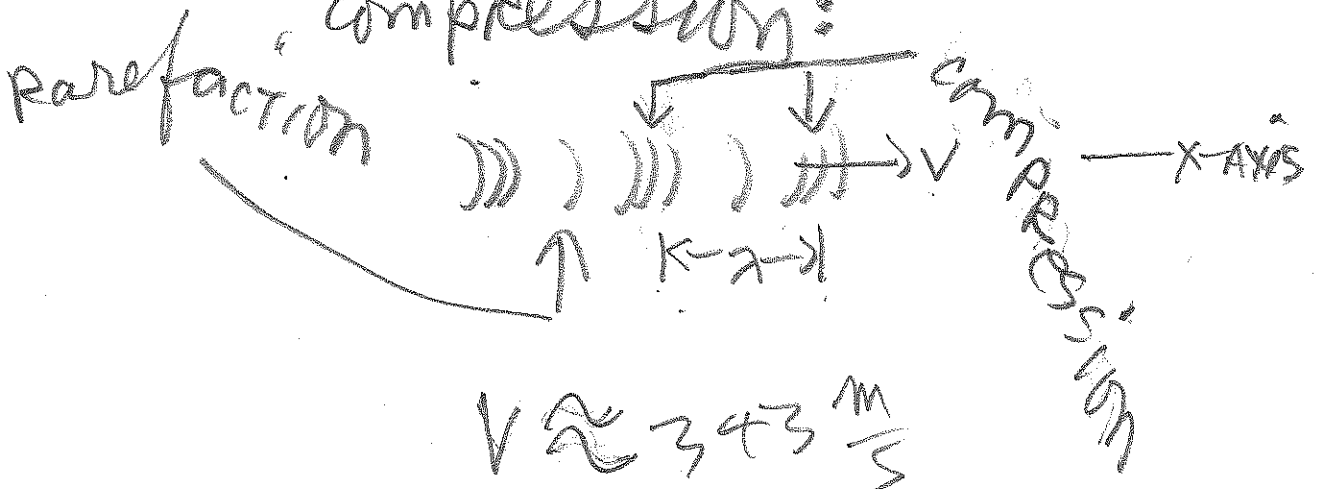
medium at x and t.

"state of displacement"

moves right; 2 special

states: rarefaction and

compression:



Derive equation for pressure p from y :
 $y_1 = y(x, t)$

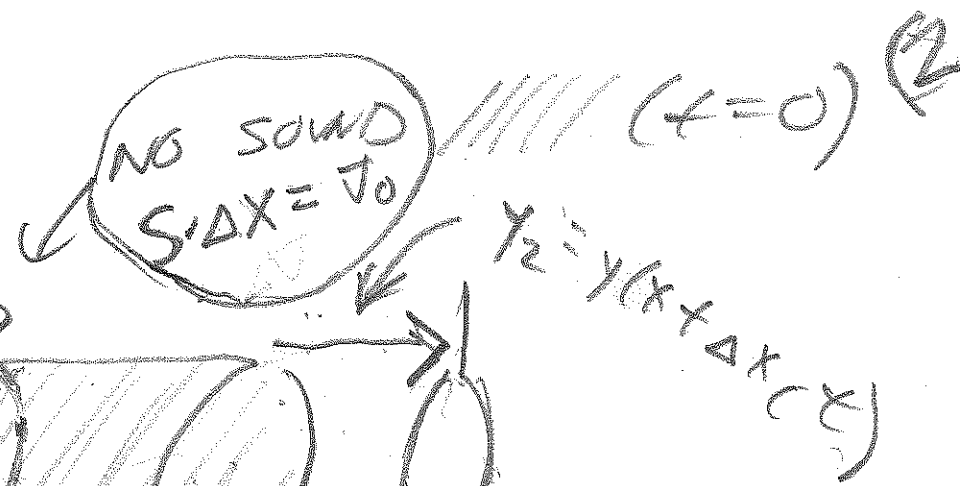
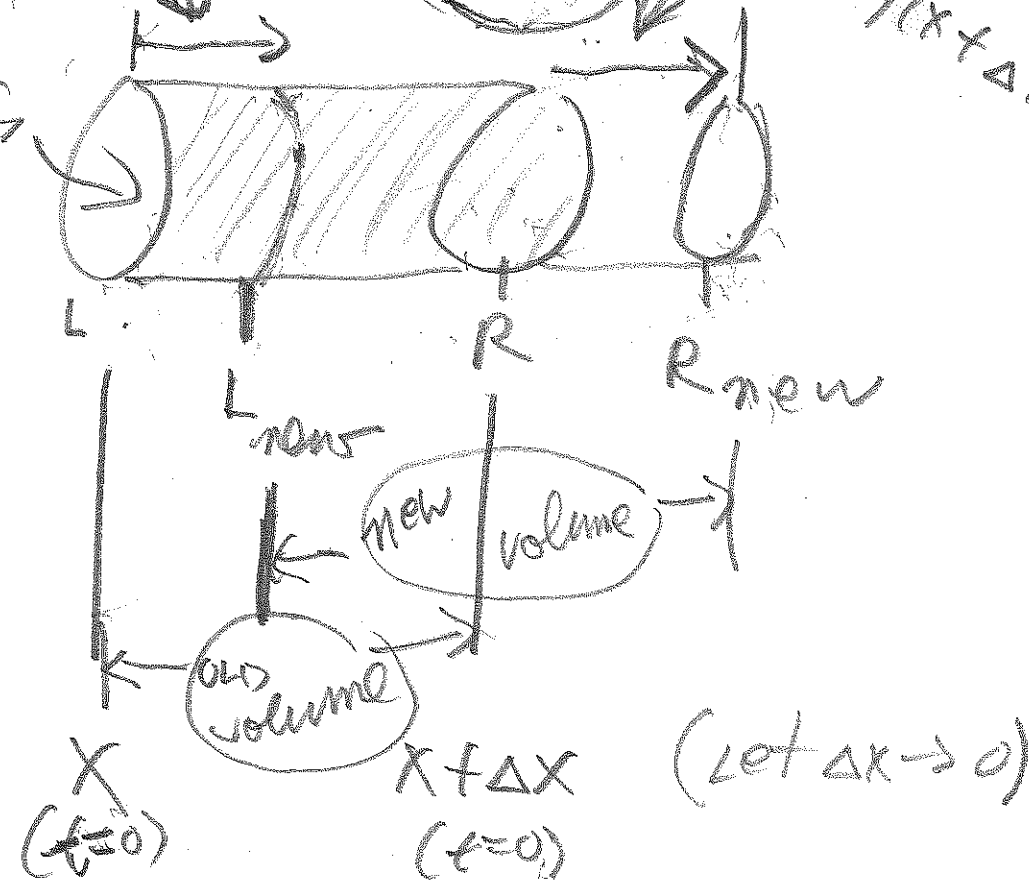


fig 16.2



ΔV is change after t

ΔV is change after dt

$y_2 < y_1$ volume decreases

" increases

$y_1 = y_2$ " same (only SHIFTS)

change in volume at time t (from $t=0$)

$$\Delta V = S_0 (y_2 - y_1) = S \cdot [y(x + \Delta x, t) - y(x, t)]$$

$$\frac{\Delta V}{V_0} = \frac{S [y(x + \Delta x, t) - y(x, t)]}{S \cdot \Delta x} \xrightarrow{\Delta x \rightarrow 0} = \frac{\partial y}{\partial x} = \frac{dV}{V_0}$$

$$\frac{dV}{V} = \frac{dV}{V}$$

3D spring constant

NOTE:
P4A

$$B = \frac{-\Delta P}{\Delta V/V_0}$$

change (fluctuation)

$$B = \frac{P(x,t)}{(dV/V)}$$

(fluctuation) IN PRESSURE

$$\Rightarrow P = -B \frac{dV}{V}$$
$$= -B \frac{dV}{dV}$$

$P(x,t)$
= fluctuation about
 P_0
↑
ambient pressure

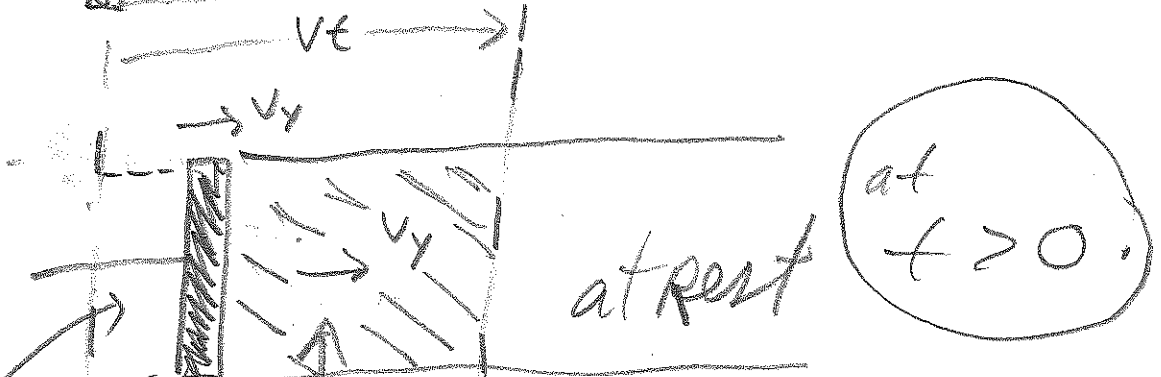
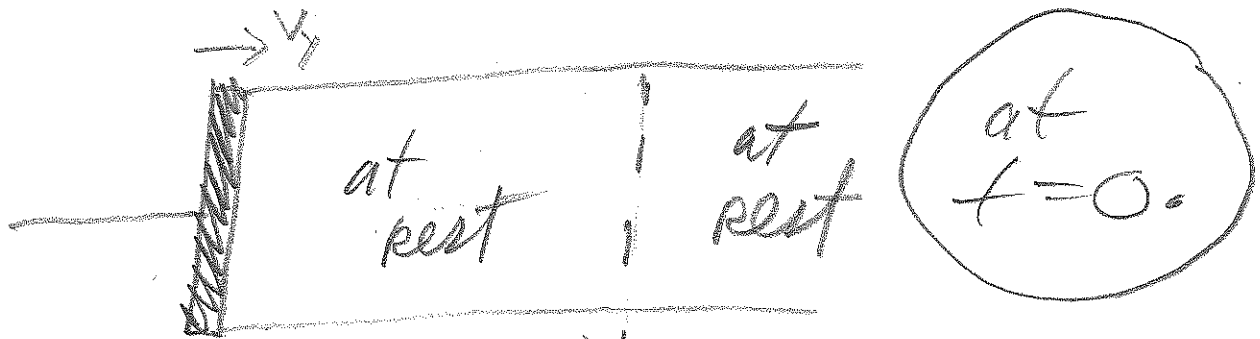
$$P(x,t) = BKA \sin(kx - \omega t)$$

↑ fluctuation

SMALL AMPLITUDE of molecule oscillation

HINT: $y \propto \cos(\dots)$
 $\frac{dy}{dt} \propto \sin(\dots)$

16.2 speed of waves (4)
 Prove $v = \sqrt{\frac{B}{\rho}}$



$$\Delta V = A \cdot v_y \cdot t$$

moving compressed fluid at time t .
 wave front of compression travels $v t$ in time t .
 $v \approx 343 \frac{m}{s}$

At $t=0$, press piston rightward with speed v_y ; continue pressing for time t .
 Longitudinal momentum of moving section:

$$\text{mass } v_y = \left(\frac{\rho \cdot A \cdot v t}{l} \right) v_y$$

$$B = \frac{-\Delta P}{\Delta V / V_0} = \frac{-\Delta P}{\frac{-A v_y t}{A \cdot v t}}$$

$$\Rightarrow \Delta P = B \cdot \frac{v_y}{v}$$

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$$\text{change in pressure} = \Delta p = B \frac{v_y}{v}$$

$F \cdot t = \text{change in momentum}$

$$\Delta p \cdot A \cdot t = (\rho v t \cdot A) v_y$$

$$B \frac{v_y}{v} \cdot A \cdot t = (\rho v t \cdot A) v_y$$

$$\frac{B}{\rho} = v^2$$

$$\sqrt{\frac{B}{\rho}} = v$$

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speed of sound in gas

$$v = \sqrt{\frac{\gamma RT}{M}}; \quad M = \text{molar mass} \quad \left(\frac{\text{kg}}{\text{mole}} \right)$$

$T = \text{temperature}$

$$R = 8.3144 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

Example $T = 293 \text{ K}$
 $= 20^\circ \text{C}$

$$\gamma = 1.4$$

$$M = 28.8 \times 10^{-3} \frac{\text{kg}}{\text{mole}} \quad (\text{AIR})$$

$$v = 349 \frac{\text{m}}{\text{s}}, \text{ as expected}$$

(7)

$$p(x,t) \cdot y(x,t) = I = \text{Intensity}$$

$$I = BkA \sin(kx - \omega t) \cdot \omega A \sin(kx - \omega t)$$

$$I = B\omega k A^2 \sin^2(kx - \omega t)$$

$$\left(\frac{\text{J}}{\text{s} \cdot \text{m}^2} \right)$$

$$I = \text{intensity} \left(\frac{\text{J}}{\text{s} \cdot \text{m}^2} = \frac{\text{Watts}}{\text{m}^2} \right)$$

$$\langle I \rangle = B\omega k A^2 \langle \sin^2(kx - \omega t) \rangle$$

$$= \frac{B\omega k A^2}{2} \left(\frac{\text{W}}{\text{m}^2} \right)$$

Intensity in decibels

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$$I = \frac{1}{2} B \omega^2 k A^2$$

$$I = \frac{1}{2} \sqrt{pB} \cdot \omega^2 A^2$$

since $\sqrt{pB} \cdot \omega = B \cdot \omega \cdot k$

OR

$$\frac{1}{v} = \sqrt{\frac{p}{B}} = \frac{k}{\omega} \rightarrow v = \sqrt{\frac{B}{p}}$$

$$I = \frac{\omega P_{max}}{2Bk} = \frac{v P_{max}}{2B} \text{ since } v = \frac{B}{p}$$

$$I = \frac{P_{max}}{2pv} = \frac{P_{max}}{2 \cdot \sqrt{pB}}$$

decibels: $\beta = (10 \text{ dB}) \log \frac{I}{I_0}$

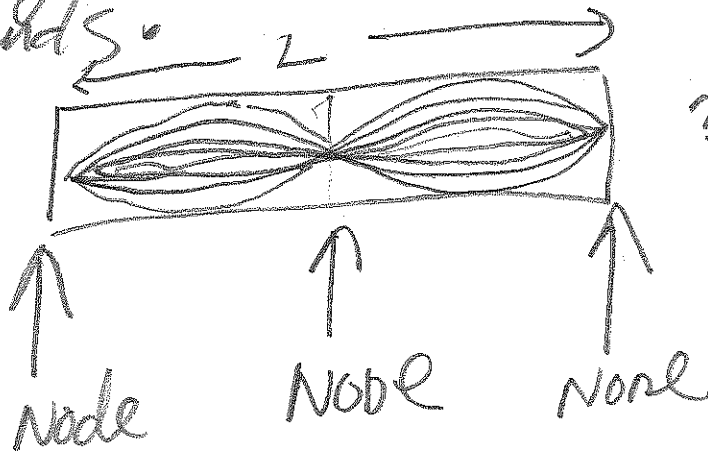
TO CONTROL VAST CHANGES IN I

NOTE: $I_0 = 10^{-12} \frac{W}{m^2}$; RADIO: $I = 10^{-8} \frac{W}{m^2}$; TRAIN: $\frac{I}{I_0} = 10^{-3} \frac{W}{m^2}$

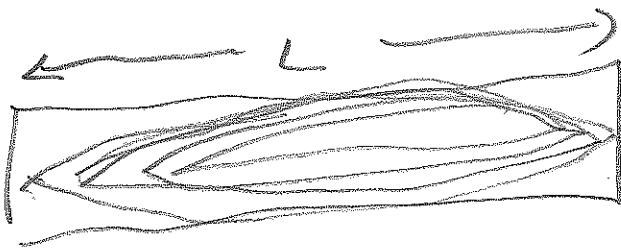
Sec 16.4

closed and open pipes

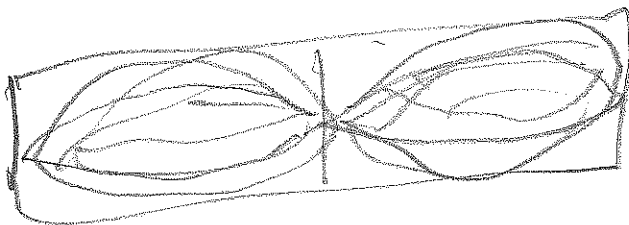
fig. 16.13; closed at both ends



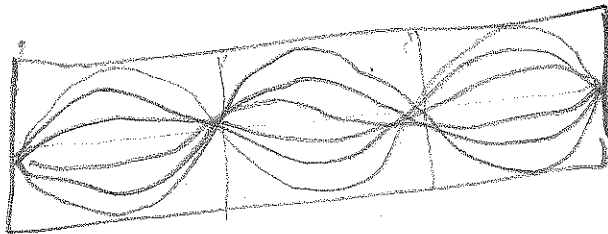
note: $L = \lambda$
 (fig. 16.13)
 ← displacement nodes



$$f_1 = \frac{(1)\nu}{2L}$$



$$f_2 = \frac{(2)\nu}{2L}$$



$$f_3 = \frac{(3)\nu}{2L}$$

Same as string!

$$f_n = \frac{n\nu}{2L}$$

$n = 1, 2, 3, 4, \dots$

(10)

Fig 16.13 :

NOTE: a displacement

node N corresponds

to a pressure ^{*}anti-node

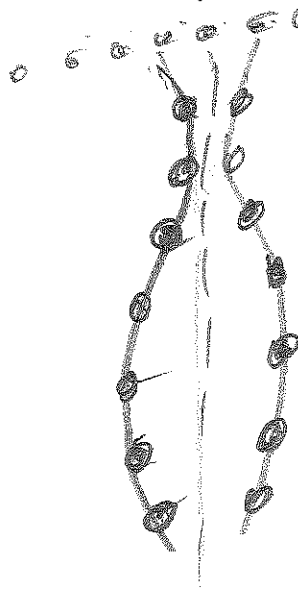
(MAXIMUM AMPLITUDE).

$N = \text{displacement}$

node (ZERO ALWAYS)

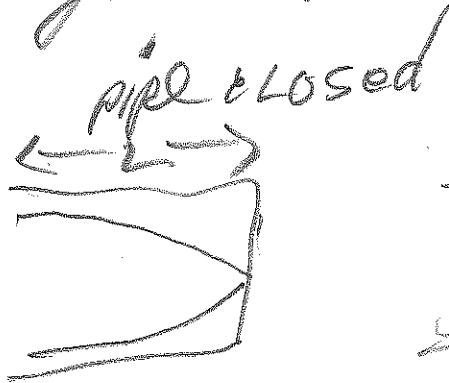
zero
amplitude

* pressure
change



A = pressure
anti node
(MAX OR MIN)

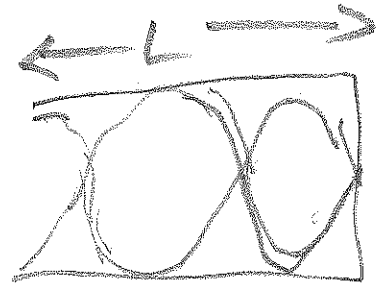
Today's Lab 16.18



$$f_1 = \frac{v}{4L}$$



$$f_3 = \frac{3v}{4L}$$



$$f_5 = \frac{5v}{4L}$$

$f_n = \frac{nL}{4L}, n=1,3,5,...$ $= 3f_1$ $= 5f_1$

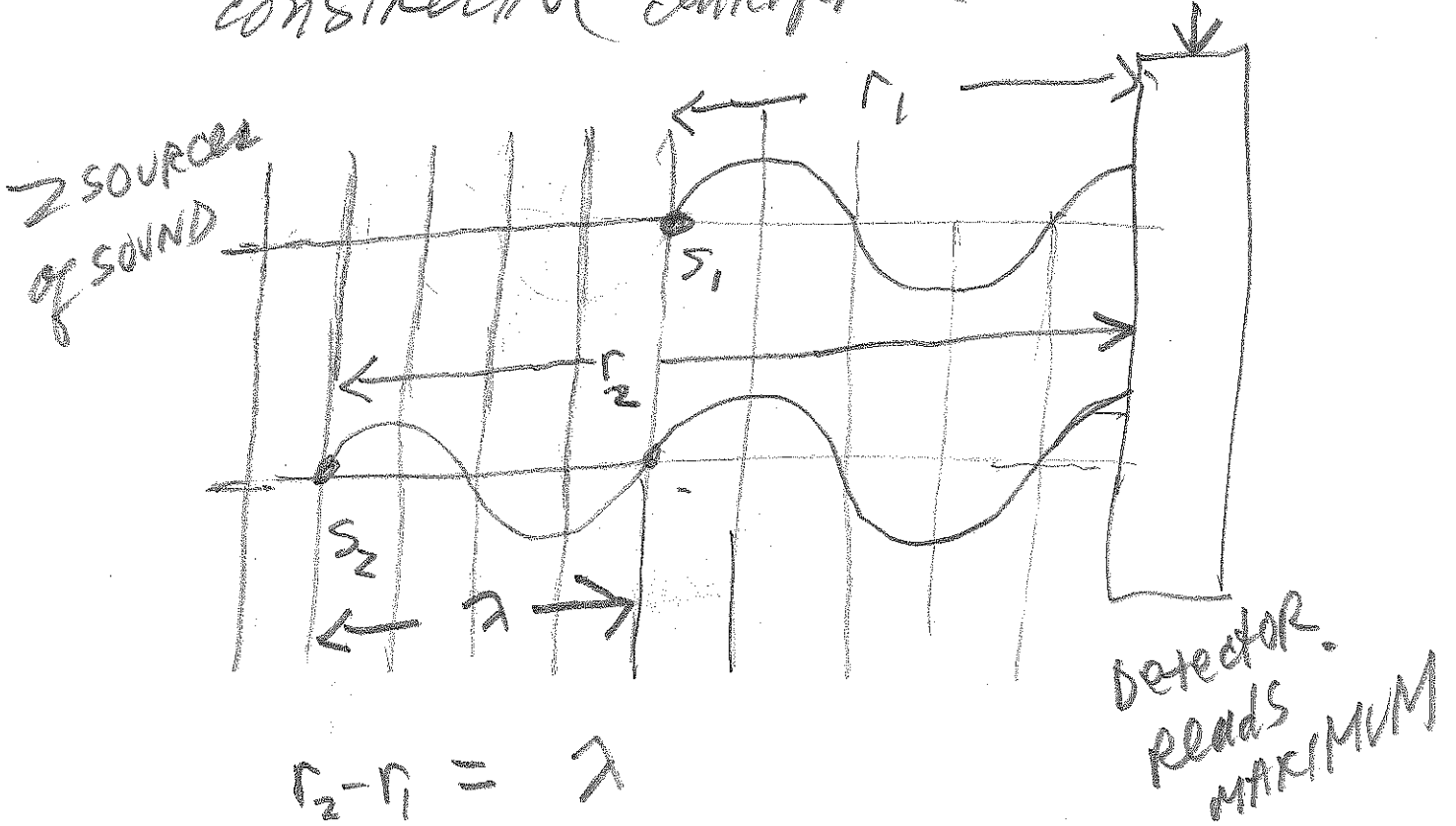
Resonance: drive system at natural frequency result in large energy absorption and amplitude.

In Lab we vary L , $f =$
constant

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Interference

constructive interference: detector



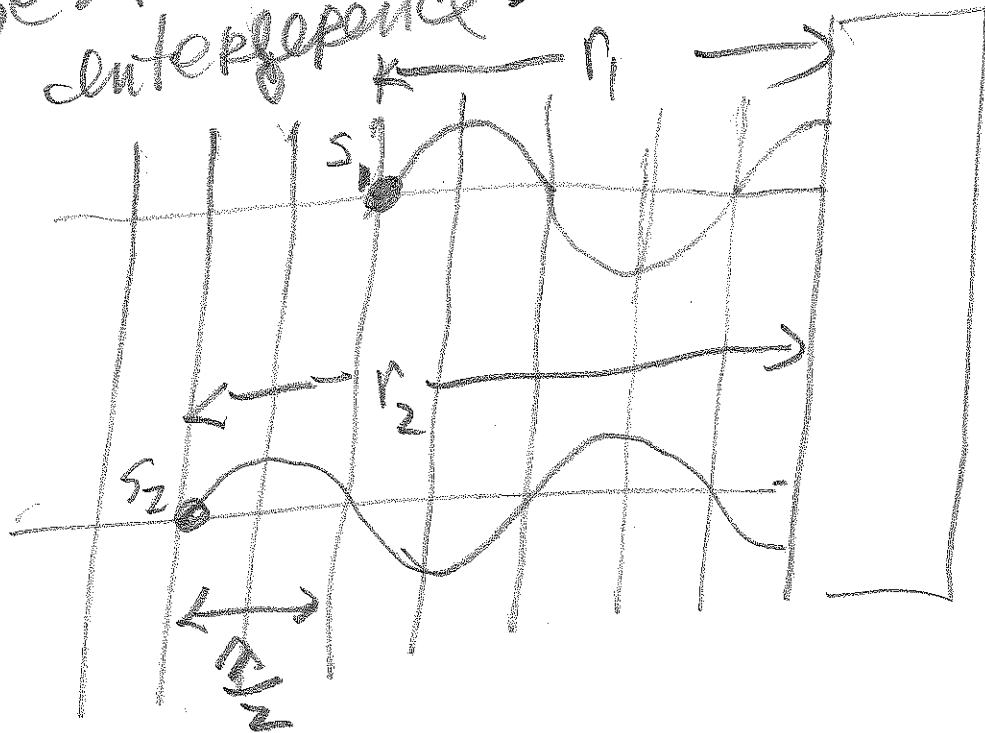
$$r_2 - r_1 = \lambda$$

In general: $|r_2 - r_1| = n\lambda,$

$$n = 0, 1, 2, 3, 4, \dots$$

↑
IN PHASE

Destructive interference = $\lambda/2$



Detector HAS ZERO Reading

$$r_2 - r_1 = \frac{\lambda}{2}; \text{ in general}$$

$$|r_2 - r_1| = (2n + 1) \cdot \frac{\lambda}{2}, n = 0, 1, 2, 3, 4, \dots$$

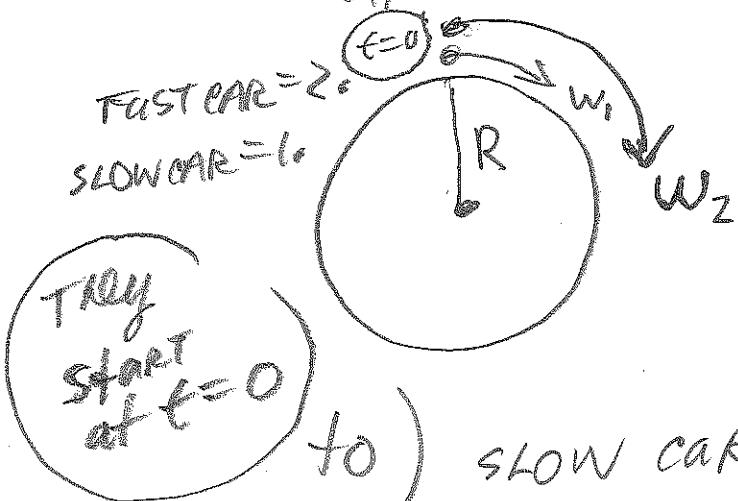
Beats

(14)

analogy:

4A problem 2 cars
 traveling at 2 different
 frequencies around a
 circular track of radius R .

FAST CAR MAKES
 1 revolution before
 being able to
 pass by (catch up)



to) SLOW CAR. TOTAL TIME TO catch up
 is $\frac{2\pi}{\omega_2} + \frac{2\pi + \omega_1}{\omega_2(\omega_2 - \omega_1)}$. Here's a

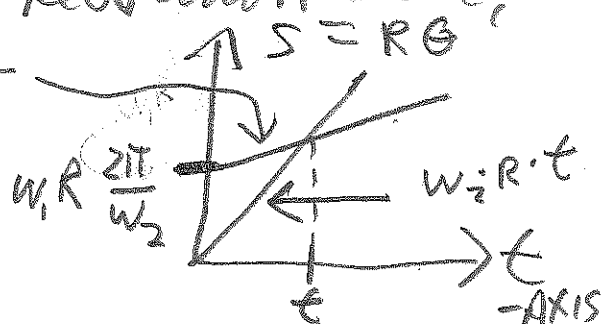
$$is \frac{2\pi}{\omega_2} + \frac{2\pi + \omega_1}{\omega_2(\omega_2 - \omega_1)}$$

proof: (A) to complete one
 revolution has time $\frac{2\pi}{\omega_2}$. (B) to

catch up after 1 revolution solve:

$$\omega_2 R \cdot t = \omega_1 \cdot R \cdot \frac{2\pi}{\omega_2} + \omega_1 R \cdot t$$

$$t = \frac{2\pi \omega_1}{\omega_2(\omega_2 - \omega_1)}$$



Let's ADD and take
reciprocal to get the
"beat" frequency.

$$\frac{2\pi}{\omega_2} + \frac{2\pi\omega_1}{\omega_2(\omega_2 - \omega_1)}$$

$$= \frac{2\pi(\omega_2 - \omega_1) + 2\pi\omega_1}{\omega_2(\omega_2 - \omega_1)}$$

$$= \frac{2\pi\omega_2}{\omega_2(\omega_2 - \omega_1)} = \frac{2\pi}{(\omega_2 - \omega_1)}$$

= Beat period.

⇒ Beat frequency = $\frac{\omega_2 - \omega_1}{2\pi}$

$$= \frac{\omega_2}{2\pi} - \frac{\omega_1}{2\pi} = \boxed{f_2 - f_1}$$

$$= f_{\text{beat}}$$