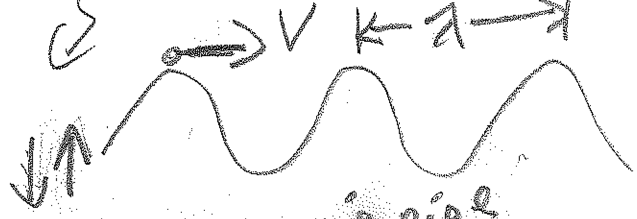
4C: 8-26-13 WAVES (contd)

CH 15

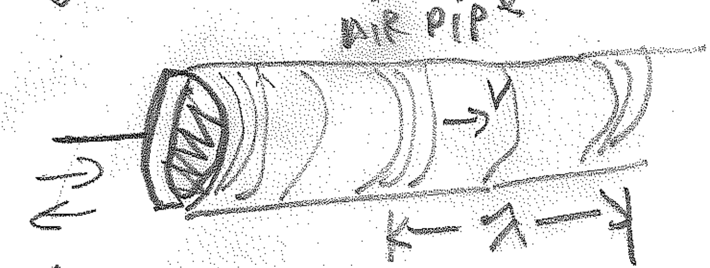
* f = frequency
force oscillation, f *

sec 15.1 Transverse



Shake a string

Longitudinal

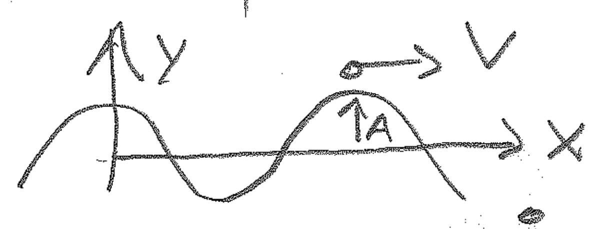


shake a piston

f = frequency

$$\lambda = V \cdot T = \frac{V}{f}, \quad T = \frac{1}{f} = \text{period}$$

math description



$$y = A \cdot \cos(kx - \omega t)$$; moves right (negative sign)

OR

$$y = A\sin(kx - \omega t)$$, A = amplitude

$$k = \frac{2\pi}{\lambda} \text{ and } \omega = \frac{2\pi}{f}$$

Note:

$$y = A\cos(kx - \omega t)$$

$$y = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$

periodic in x with:

$$y(x,t) = y(x+\lambda, t)$$

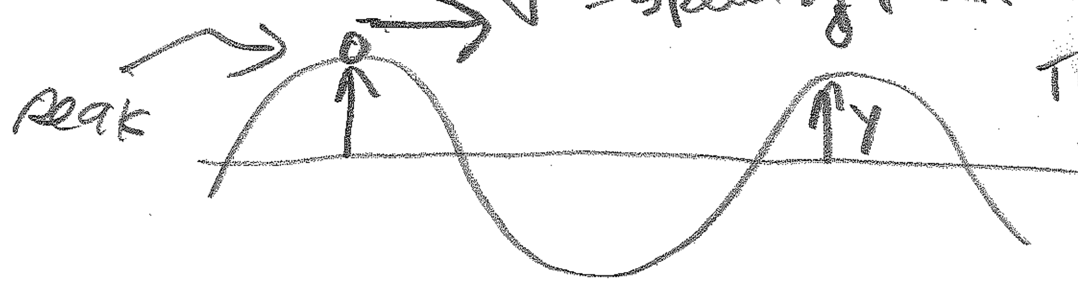
and periodic in T:

$$y(x,t) = y(x, t+T)$$

wave speed: $v = ?$

Let $y = A\cos(kx - \omega t) = \text{constant}$

$\rightarrow v$ = speed of peak (along x);



Transverse wave.

"RIDE" with WAVE.

$(kx-\omega t) = \text{constant}$

$kx - \omega t = \text{constant}$

$$\frac{d(kx-\omega t)}{dt} = 0$$

$$k\frac{dx}{dt} - \omega\cdot\frac{dt}{dt} = 0$$

WAVE MOVES left:

$kx + \omega t = \text{constant}$

$v_x = -\frac{\omega}{k}$

left motion.

$v_x = dx/dt$

$$k\cdot v_x = \omega$$

$$v_x = \frac{\omega}{k} = \text{peak (wave) velocity}$$

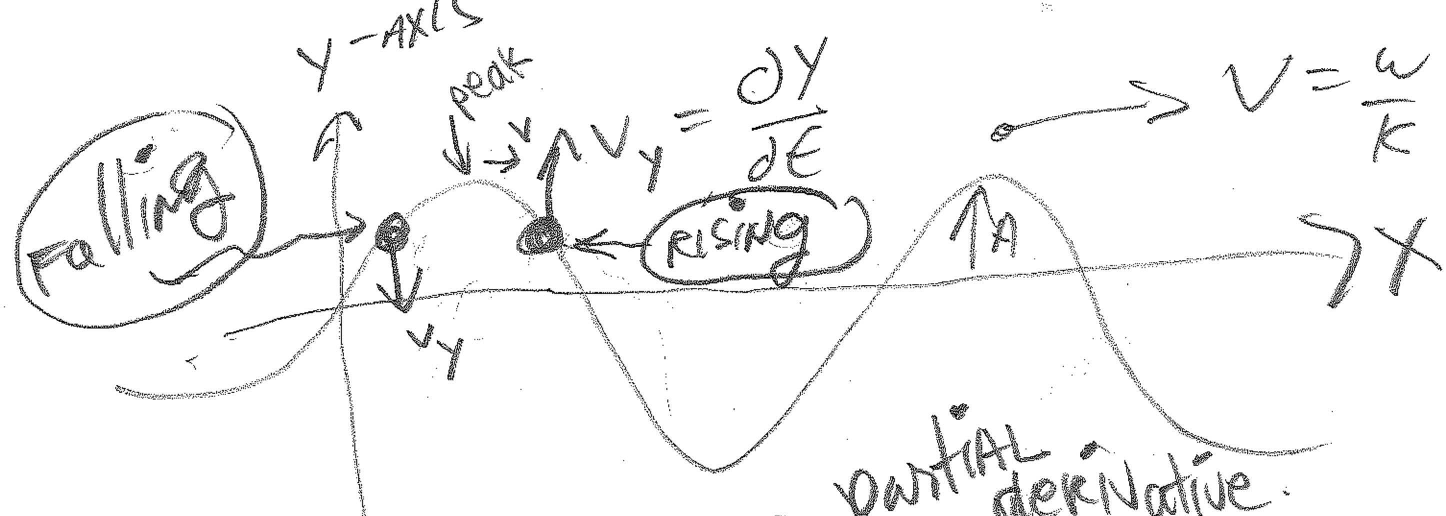
$Y = A\cos(kx-\omega t)$; $v_x > 0$

negative when waves move right.

TRANSVERSE VELOCITY: $v_y = \dfrac{\partial y}{\partial t}$

NOTE: $\dfrac{\partial y}{\partial t}$ = PARTIAL DERIVATIVE RELATIVE TO t. (PRETEND x = CONSTANT.)

$y = A\cos(kx - \omega t)$ ← WAVE MOVES RIGHT



← partial derivative

$$v_y = \frac{\partial y}{\partial t} = +A\omega\sin(kx - \omega t)$$
(LET x = CONSTANT)

$$a_y = \frac{\partial^2 y}{\partial t^2} = A\omega^2\cos(kx - \omega t) = -\omega^2 y$$

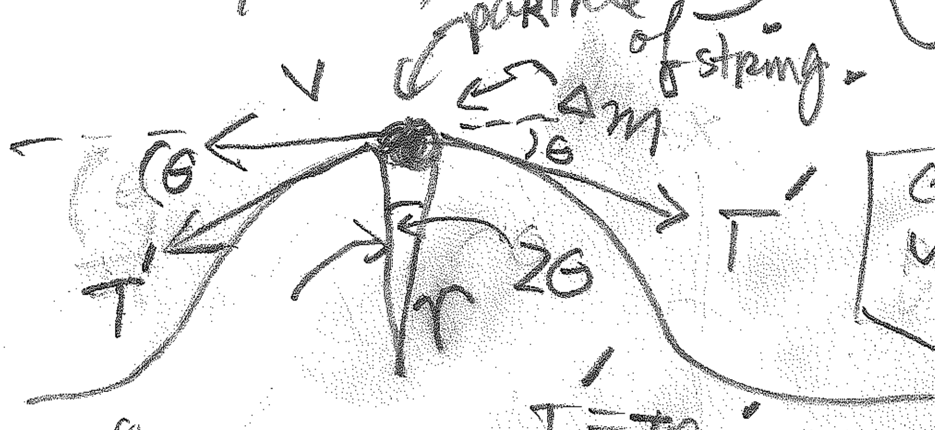
NOTE, WAVE EQUATION: $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$; $v^2 = \dfrac{\omega^2}{k^2}$

ASSUME: $y = A\cos(kx - \omega t)$

<u>HINT</u>: Math 3
Partial differentiation

Sec 15.4 speed of transverse wave
<u>NOT IN BOOK.</u>

my way:



GO TO FRAME with speed v relative TO GROUND

T' = Tension

$$\frac{\Delta m v^2}{r} = 2T' \sin\theta$$

μ = mass density $\left(\frac{kg}{m}\right)$

$\sin\theta \simeq \theta \ll 1$

$$\Delta m = \mu \cdot \Delta x = \mu \cdot r \cdot 2\theta$$

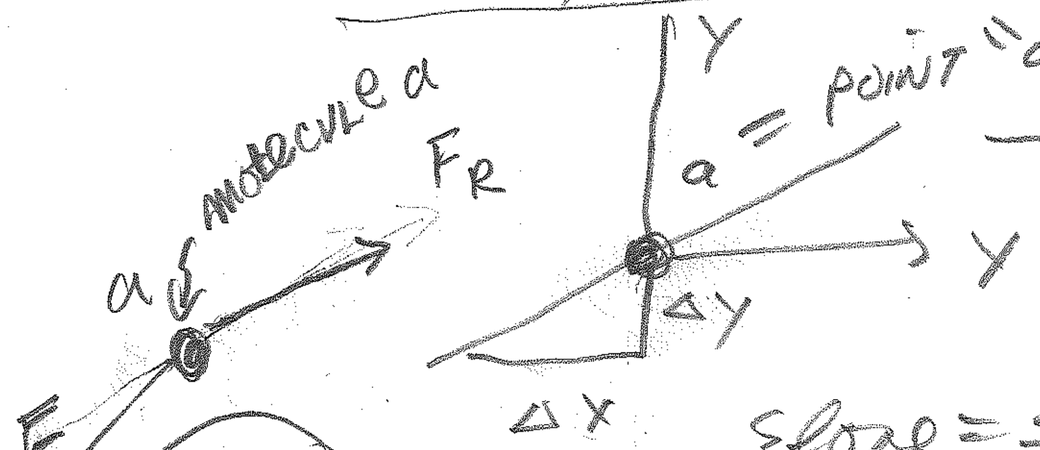
T' = tension $= F$

$$\frac{\mu r \cdot 2\theta}{r} v^2 = 2T'\theta$$

$$\longrightarrow v = \sqrt{\frac{T'}{\mu}} = \sqrt{\frac{F}{\mu}}$$

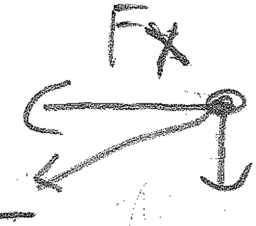
NOTE: $V = \sqrt{\dfrac{\text{Elastic quality}}{\text{Inertial quality}}}$

Energy in Waves



$\text{slope} = \dfrac{\Delta Y}{\Delta X}$

NOTE: $|F_{Ry}| > |F_{Ly}|$



⇐ Force from string to left of "a".

$F_L = F$

$$F_Y = -F \cdot \frac{\partial Y}{\partial X} \approx -F \cdot \frac{\Delta Y}{\Delta X}$$

Force on "a" from string to left.

$P_{(x,t)}$ = Power delivered to "a" by string to left.

P4A

$$P \equiv F_y \cdot V_y = -F \frac{\partial Y}{\partial X} \cdot \frac{\partial Y}{\partial t}$$

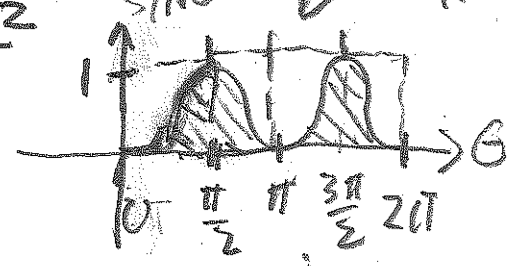
$$\frac{\partial Y}{\partial X} = -KA\sin(kx-\omega t) \text{ and } \frac{\partial Y}{\partial t} = \omega A\sin(kx-\omega t)$$

$$\underline{\text{Thus:}}\ P = F \cdot k \cdot \omega A^2 \sin^2(kx-\omega t) \quad (\text{MATH3})$$

$$F = \text{tension} = T$$

4 SECTIONS EQUAL AREA

$$\boxed{\tfrac{1}{2}} = \langle \sin^2\theta \rangle = \frac{\text{AREA}}{2\pi} = \frac{2\pi \cdot 1/2}{2\pi}$$



AVERAGE OVER TIME

AVERAGE POWER $= \langle P \rangle$

$$= Fk\omega A^2 \langle \sin^2(kx-\omega t) \rangle$$

$$\frac{W}{K} = \sqrt{\frac{F}{M}}$$

$$F \cdot K^2 \sqrt{\frac{F}{M}} \cdot A^2$$

$$= F \cdot k\omega \cdot A^2 \cdot \frac{1}{2}$$

$$= \frac{1}{2} F \cdot k\omega A^2 \text{; use } W = V \cdot K \text{ \& } V^2 = \frac{F}{M}$$

$$= \underline{\frac{\sqrt{MF}}{2} \cdot W^2 \cdot A^2} \text{; } F = T = \text{tension}$$

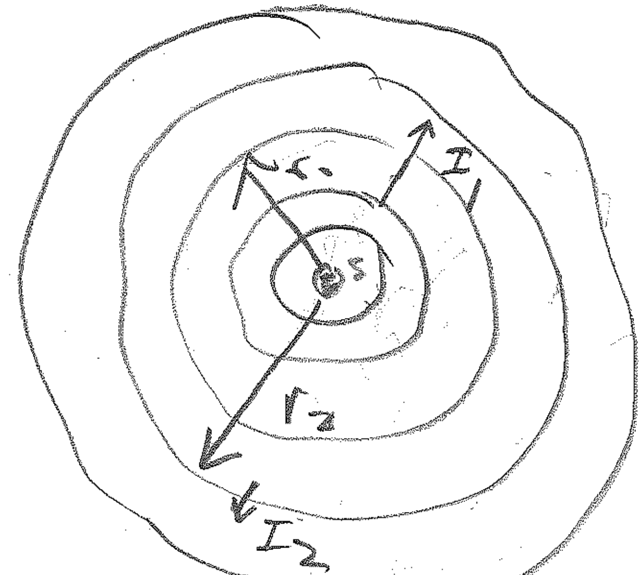
using substitutions above.

$$\boxed{\begin{aligned} & F \cdot k\omega A^2 \\ &= F \cdot k \cdot V \cdot k \cdot A^2 \\ &= F \cdot k^2 \sqrt{\frac{F}{M}} \cdot A^2 \\ &= F \cdot \frac{W^2}{V^2} \sqrt{\frac{F}{M}} \cdot A^2 \\ &= \sqrt{MF} \cdot W^2 \cdot A^2 \end{aligned}}$$

Power on a string delivered to some point "a"

Intensity (sec 15.5)

3D



Intensity:

$$I = \frac{\text{Power}}{\text{area}}$$

$$I = \frac{P}{4\pi r^2}$$

"spherical" waves

$$\text{Power} = 4\pi r_1^2 \cdot I_1 = 4\pi r_2^2 \cdot I_2$$

$$\boxed{\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}}$$

(15.6) Boundary conditions and superposition

(15.7) standing waves = superposition of 2 waves moving in opposite directions