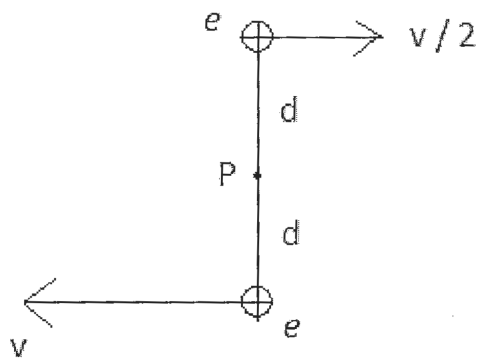
solutions posted below:

Throughout this examination the symbol for out is $\odot$. ***The symbol for in is*** $\otimes$. ***SHOW ALL WORK.***

1. (40 points) At some instant, two protons on "try out" at CERN's epochal experiment move in *opposite* directions ***horizontally.*** They travel relative to stationary midpoint P. The distance d is 8.00×10^{-15} m -- on the order of a typical nuclear separation. The proton charge is $e = 1.6\times10^{-19}$ C. The upper proton moves half as fast as the *lower* one, which has non-relativistic speed $v = 9.00\times10^{6}$ m/s.



(a) (30 points) When the two charges are at the *vertically displaced* locations shown in the above figure, what are the DIRECTION AND MAGNITUDE of the NET magnetic *field* they produce at P?

(b) (4 points) What is the direction and magnitude of the magnetic *force* on the upper proton?

(c) (2 points) What is the direction and magnitude of the magnetic *force* on the lower proton?

(d) (4 points) If the *upper* proton's direction of motion were reversed, so both charges were moving in the same direction, what would be the direction and magnitude of the magnetic *force* on the upper proton? What's the direction and magnitude of the magnetic force on lower proton in that case?

(e) EXTRA CREDIT (4 POINTS) Using symbols, including k and μ_o, find the ratio of the magnitudes of the magnetic *force* over the electric *force* on *upper* proton. Which force has the larger magnitude, the electric or magnetic?

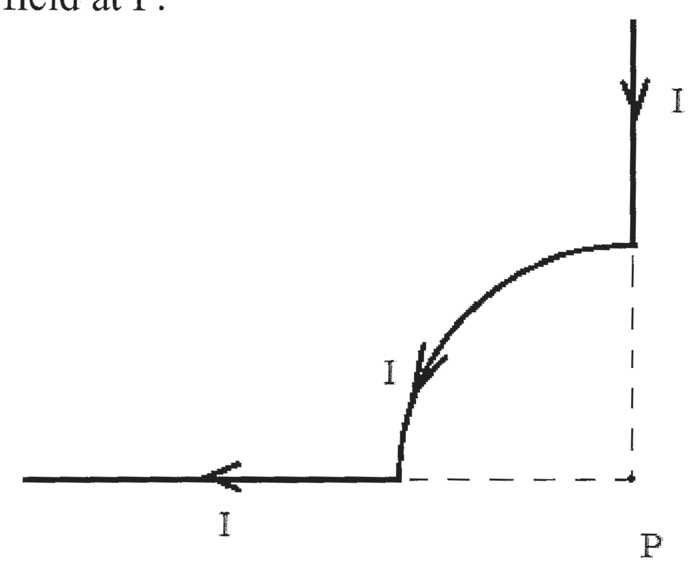
2. (40 points) This problem deals with a sensitive current sensor based on detection of a magnetic field at a central location. Use symbols. Find the

(a) (10 points) direction and

(b) (26 points) magnitude

of this magnetic field at point P due to the quarter-circular section of wire with current I shown in the schematic of sensor. The circular-arc section has radius R and point P is at *the center*.

(c) (4 points) Using and explaining a formula from Ch. 28, show that current I in the long *straight* sections of the wire produce *no* field at P.



3. (18 points) Current sensor based on a magnetic field due to *straight* wire section. A conductor of length 2L carries a current I.

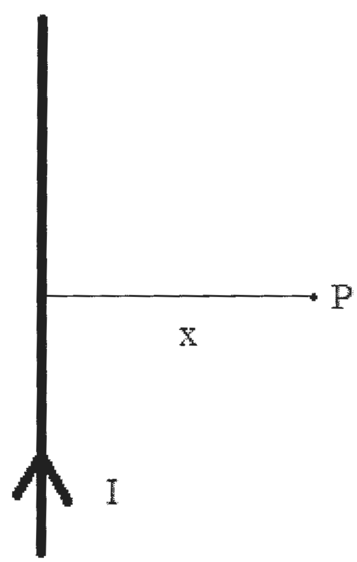
Using symbols, find

(a) (6 points) The direction and

(b) (8 points) magnitude

of the magnetic *field* at point P a distance x from the conductor on its perpendicular *bisector*.

(c) (4 points) What is the answer to part (b) in the limit that L goes to infinity while distance x is held constant?



4. (40 points) Inside a damaged control panel on the Star Federation Ship Enterprise, two horizontally displaced long parallel wires have currents flowing as shown. The wires are *perpendicular* to the page. The distance between the wires is $d = 0.035$ m. The current I_A in wire A is 9.00 (A) OUT. The current I_B in wire B is 9.50 (A) IN. Figure 2 is for part (d).

Note: $\mu_o = 1.257 \times 10^{-6}$ T·m/A = $4\pi 10^{-7}$ T·m/A. For parts (a), (b), (c) see figure 1; for part (d), figure 2.

(a) (20 points) What is the *magnitude* of the *net* magnetic field due to *both* wires at *midpoint* P of the perpendicular segment between wires?

(b) (10 points) What is the *direction* of the net magnetic field due to both wires at midpoint P?

(c) (6 points) What is the magnitude and direction of the magnetic *force* on a 1.00 m section of wire B due to the magnetic field of wire A?

(d) (4 points) In figure 2, a point particle with *negative* charge q at midpoint P between wires moves with velocity OUT OF PAGE . What is the (i) *direction and* (ii) *magnitude* of the force on the charge at that moment, assuming $q = -1.4 \times 10^{-6}$ (C) and speed $v = 2.00 \times 10^{6}$ m/s.

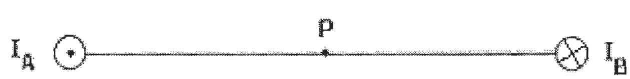


figure 1

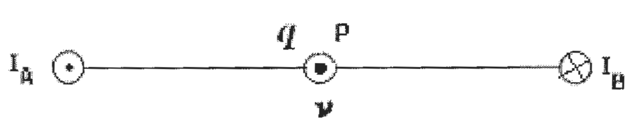


figure 2

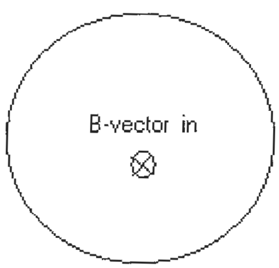
5. (35 points) A single circular loop of radius r = 1.0 m is placed in a region where a uniform magnetic field is perpendicular to the loop's plane. The magnetic field vector points <u>in</u>. While the vector points <u>in</u>, the magnitude *B* of the magnetic field is allowed to change linearly with time t according to the equation:
$B = B_0 - 0.0150 \cdot t$ (T), where time t is in seconds. Assume at time t the expression for B is positive and the vector $\vec{B}$ points in. B_0 is the value of the changing magnitude B at time t = 0.

(a) (10 points) What is the <u>direction</u> of the induced current in the loop, clockwise or counterclockwise, for $t \geq 0$? Please indicate this direction by drawing an arrow at the wire. Explain your reasoning.

(b) (10 points) What is the <u>direction</u> of the induced magnetic field vector $\vec{B}_{IND}$, *in* or *out*, for $t \geq 0$? Please indicate this direction by drawing a symbol within the loop boundary. Explain your reasoning.

(c) (10 points) Calculate the magnitude $|\varepsilon|$ of the induced emf (in volts) for $t \geq 0$.

(d) (5 points) Suppose the loop has a resistance R = 800.00 Ω. What would be the value of the induced current I for $t \geq 0$?



wire loop

6. EXTRA CREDIT. 12 POINTS. LR series circuit with current *build up* . An inductor with an inductance L = 2.50 H and a resistor with R = 8.00 ohms are connected in *series* to the terminals of a battery with an emf of 6.00 (V). The switch connecting these three series elements is closed at t = 0 and the current begins to increase from its initial value of zero. Find:

(a) (3) the initial rate of change of the current in the circuit.

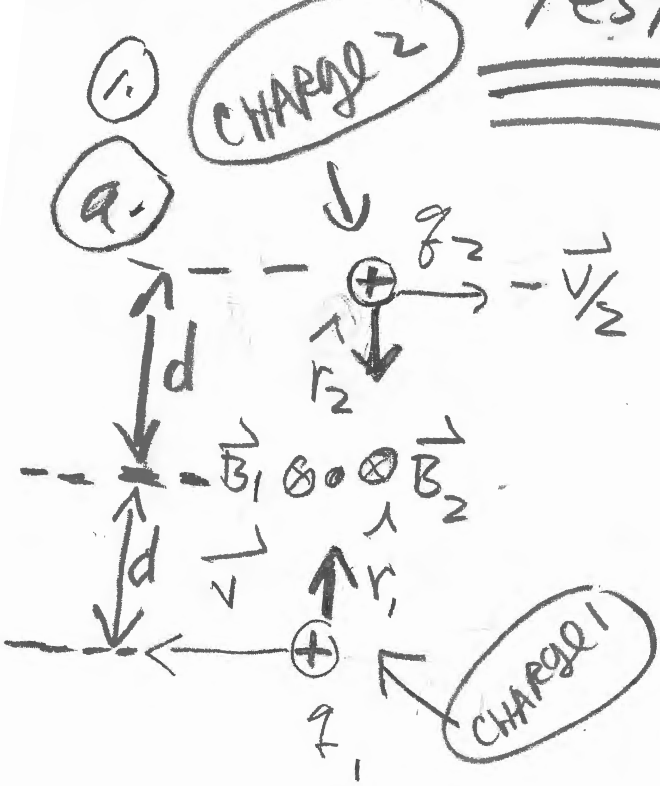
(b) (3) the rate of change of the current when the current is 0.500 (A)

(c) (3) the current 0.250 seconds after the switch is closed

(d) (3) the final steady state current after a very long time.

Test4 Solutions

1. a.



⊗ $\vec B_1 = \frac{\mu_0}{4\pi}\cdot\frac{q\vec v\times\hat r_1}{r^2}$ IN

⊗ $\vec B_2 = -\frac{\mu_0}{8\pi}\cdot\frac{q\vec v\times\hat r_2}{r^2}$ IN

Note: $\hat r_2 = -\hat r_1$.

$\vec B_{NET} = \vec B_1 + \vec B_1$ at P

$$\underbrace{\tfrac{2}{2}+\tfrac{1}{2}}$$

$$= \frac{3}{2}\cdot\frac{\mu_0}{4\pi}\cdot\frac{q\vec v\times\hat r_1}{r^2} \quad \text{IN}$$

$$|\vec B_{Net}| = B_{net} = \frac{3\mu_0 q V\sin 90^0}{8\pi d^2}$$

$$B_{net} = \frac{3\mu_0}{8\pi}\cdot\frac{eV}{d^2}$$

$$\frac{3\cdot(4\pi\times10^{-7})}{8\pi}\cdot\frac{1.6\times10^{-19}\cdot9\times10^{6}}{(8\times10^{-15})^2} =$$

$$= \frac{3\times10^{-7}\cdot1.6\times10^{-19}\cdot9\times10^{6}}{128\times10^{-30}}$$

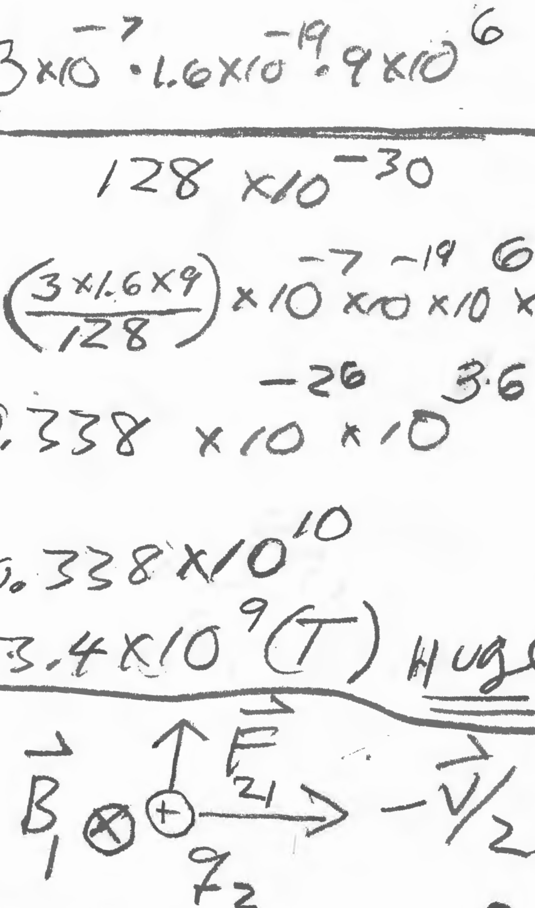
$$= \left(\frac{3\times1.6\times9}{128}\right)\times10^{-7}\times10^{-19}\times10^{6}\times10^{30}$$

$$= 0.338\times10^{-26}\times10^{36}$$

$$\cong 0.338\times10^{10}$$

$$= 3.4\times10^{9}\,(T)$$ Huge!

b.



$\vec F_{21}$ is up, $\hat j$; $|\vec F_{21}| = F_{21} = e\cdot\frac{v}{2}\cdot B_1\sin 90^0$

Note: $B_1 = \frac{\mu_0}{4\pi}\frac{eV}{(2d)^2} = \frac{1}{4}\frac{\mu_0}{4\pi}\cdot\frac{eV}{d^2}$

In PART (a): $B_1 = \frac{\mu_0}{4\pi}\cdot\frac{eV}{d^2}$ at MID-POINT.

b.) $B_1 = \frac{1}{4}(B_1 \text{ PART (a)})$ is TRUE.

$B_1 \text{ PART (a)} = \frac{\mu_0}{4\pi}\cdot\frac{eV}{d^2} = \frac{2}{3}\cdot B_{NET}$

$B_1 \text{ PART (a)} = \frac{2}{3}\cdot(0.338\times 10^{10})$

since the lower proton contributes 2 units out of 3 units of $\frac{V}{2}$.

$B_1 \text{ PART (a)} = 0.225\times 10^{10} = 2.25\times 10^{9}\ (T)$

$\rightarrow B_1 \text{ PART (b)} = \frac{1}{4}(B_1 \text{ PART (a)})$

$B_1 = 0.563\times 10^{9}\ (T).$

$F_{21} = e\frac{V}{2}\cdot B_1 = 1.6\times 10^{-19}\times \frac{9}{2}\times 10^{6}\cdot B_1$

$F_{21} = (1.6\times 10^{-19})(4.5\times 10^{6})(0.563\times 10^{9})$

$F_{21} = 4.05\times 10^{-4}\ (N)$

c.) We verify Newton's 3rd law. Note: $F_{21} = e\cdot\frac{V}{2}\cdot B_1 = e\cdot\frac{V}{2}\cdot\frac{\mu_0}{4\pi}\frac{eV}{(2d)^2}$

$= \frac{e^2\cdot V^2\cdot\mu_0}{32\pi d^2}$, in pure symbols.

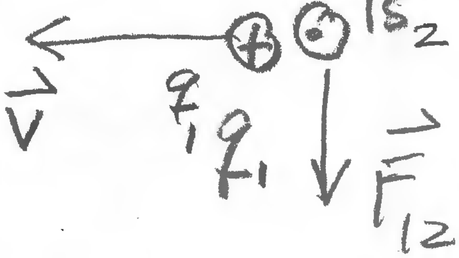
$\rightarrow \vec{F}_{21} = \frac{\mu_0 e^2 v^2}{32\pi d^2}$, up.

Let's show $\vec{F}_{12} = -\vec{F}_{21}$.

2 (c). We show: ✓

$\vec{F}_{21} = -\vec{F}_{12}$

First, let verify opposite direction:



is down.

Equal magnitude verification:

$F_{12} = eVB_2 \sin 90^\circ$

$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{e\cdot[\frac{V}{2}]}{(2d)^2}$

$B_2 = \frac{\mu_0}{32\pi}\cdot\frac{eV}{d^2}$

$F_{12} = e\cdot V\cdot B_2$

$F_{12} = eV\cdot\frac{\mu_0}{32\pi}\cdot\frac{eV}{d^2}$

$F_{12} = \frac{\mu_0 e^2 v^2}{32\pi d^2}$

$F_{12} = F_{21}$ QED

(4)

(e) $\frac{\mu_0}{32\pi}\frac{e^2v^2}{d^2}$

= MAGNETIC FORCE.

ELECTRIC FORCE $= \frac{ke^2}{4d^2}$

(SEPARATION $= 2d \to 4d^2$.

RATIO:

$$\frac{\frac{\mu_0}{32\pi}\cdot\frac{\cancel{e^2}v^2}{\cancel{d^2}}}{\frac{k\cancel{e^2}}{\cancel{4d^2}}} = \boxed{\frac{\mu_0}{k}\cdot\frac{v^2}{8\pi}}$$

$k = \frac{1}{4\pi\varepsilon_0}$

$\Rightarrow$ RATIO $= \frac{\mu_0}{\left(\frac{1}{4\pi\varepsilon_0}\right)}\cdot\frac{v^2}{8\pi}$

RATIO $= \frac{\mu_0\varepsilon_0 \cdot v^2}{2}$

NOTE: $c^2 = \frac{1}{\mu_0\varepsilon_0}$

FACT FROM PHYSICS 4C

WHERE $c = 3\times10^8\ \frac{m}{s}$

$\Rightarrow$ RATIO $= \frac{v^2}{2c^2} \ll 1$

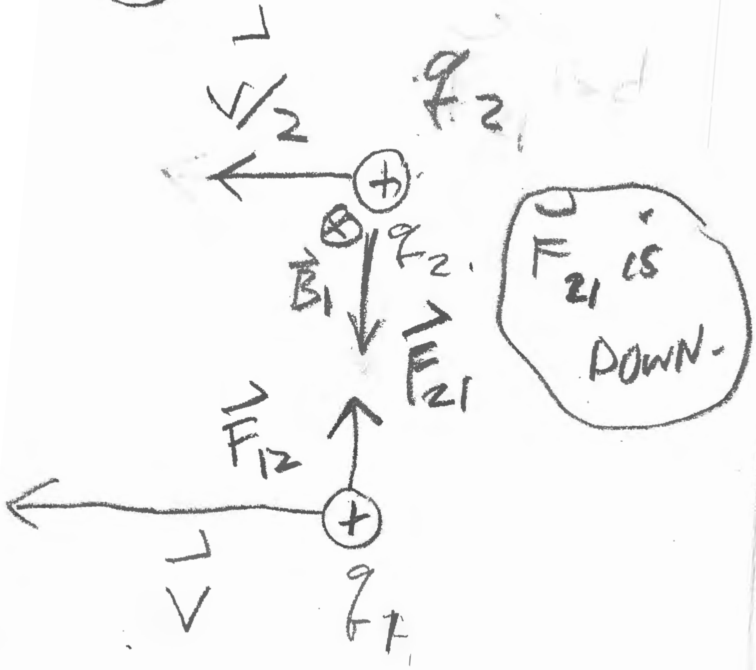
SINCE $v \ll c$.

$\Rightarrow \frac{F_B}{F_E} \ll 1$.

$\frac{F_B}{F_E} = \frac{(9\times10^6)^2}{2(3\times10^8)^2} = 1.5\times10^{-4}$

(SMALL)

(d.)



$$\vec{F}_{21} = e\frac{v}{2}\cdot B_1 \sin 90^\circ$$

$$= e\frac{v}{2}\cdot\left(\frac{\mu_0}{16\pi}\cdot\frac{ev}{d^2}\right)$$

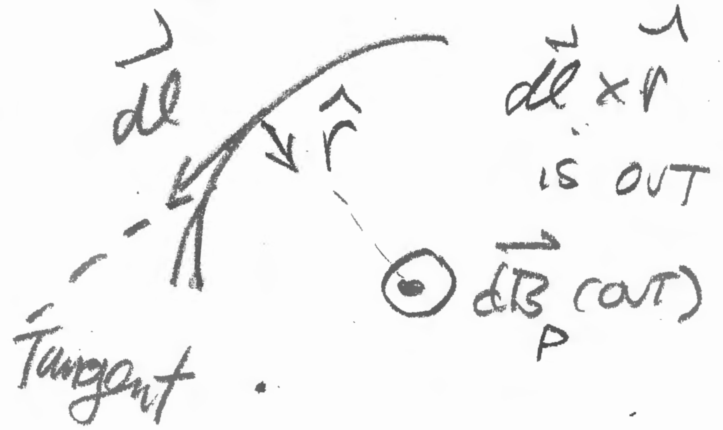
$$= \frac{\mu_0}{32\pi}\cdot\frac{e^2v^2}{d^2}$$

$F_{12} = F_{21}$.

$\vec{F}_{12}$ is up, opposite $\vec{F}_{21}$

(2.)

(a.) $\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\hat{r}}{r^2}$



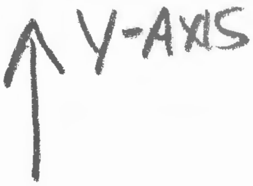
Integration preserves the direction: $\vec{B}_P$ at P is OUT.

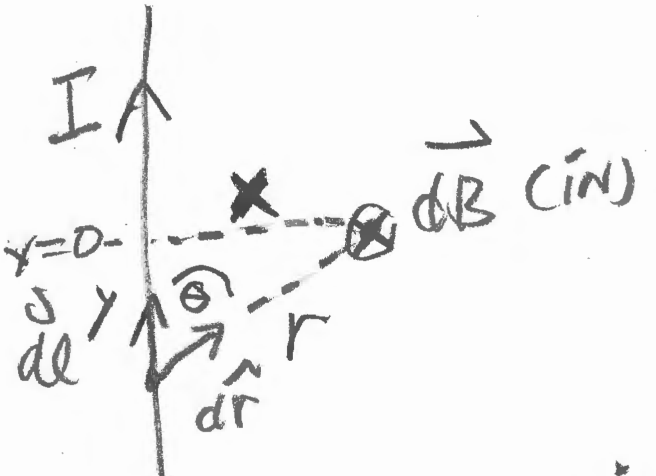
(b.)

$$dB_P = \frac{\mu_0}{4\pi}\cdot\frac{I\,dS}{R^2}$$

$$B_P = \frac{\mu_0}{4\pi}\cdot\frac{I}{R^2}\cdot\int dS = \frac{\mu_0}{4\pi}\cdot\frac{I}{R^2}\cdot\frac{\pi}{2}\cdot R$$

$\vec{B}_P = \mu_0 I/8R$, OUT.





$$|\vec{dB}| = dB = \frac{\mu_0}{4\pi}\cdot\frac{I\cdot dy\sin\theta}{r^2}$$

$$\tan\theta = \frac{x}{y} = \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \frac{y}{x} = -\cot\theta$$

- Note we use a negative sign since we will integrate between $y = y_{MIN} < 0$ and $y = 0$.

We will integrate from y_{MIN} to $y=0$ and multiply the result by 2 due to symmetry:

$$y = -x\cdot\cot\theta$$

$$dy = -x\cdot(-\csc^2\theta)\,d\theta$$

$$dy = +x\csc^2\theta\,d\theta > 0$$

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{I\cdot x\cdot\csc^2\theta\,d\theta\sin\theta}{x^2\csc^2\theta}$$

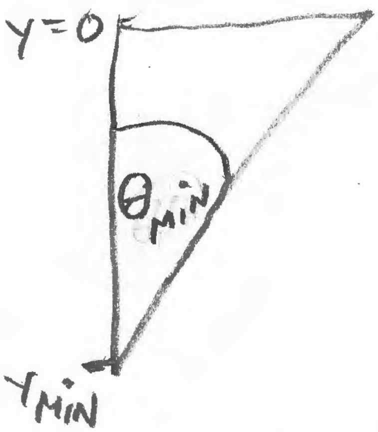
since $r = x\cdot\csc\theta$

because $\frac{x}{r} = \sin\theta$

and $\csc\theta = \frac{r}{x}$.

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{I}{x}\cdot\sin\theta\,d\theta.$$

$$B = 2\cdot\frac{\mu_0}{4\pi}\cdot\frac{I}{x}\int_{\theta_{MIN}}^{\pi/2}\sin\theta\,d\theta$$

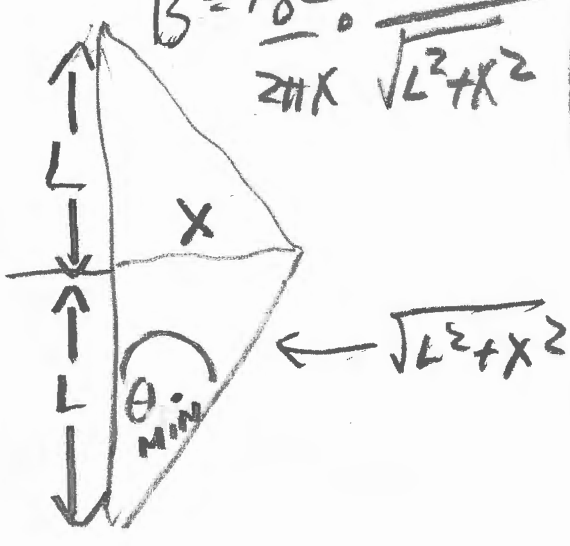


$$B = \frac{\mu_0 I}{2\pi X} \cdot \Big[-\cos\theta\Big]_{\theta_{MIN}}^{\pi/2}$$

$$B = \frac{\mu_0 I}{2\pi X} \cdot \Big[-\cos\frac{\pi}{2} + \cos\theta_{MIN}\Big]$$

$$B = \frac{\mu_0 I}{2\pi X} \cdot \cos\theta_{MIN}$$

$$B = \frac{\mu_0 I}{2\pi X} \cdot \frac{L}{\sqrt{L^2+X^2}}$$



(c) as $L \to \infty$

$$B = \frac{\mu_0 I}{2\pi X} \cdot \lim_{L\to\infty}\left[\frac{L}{\sqrt{L^2+X^2}}\right]$$

$$B = \frac{\mu_0 I}{2\pi X} \cdot 1$$

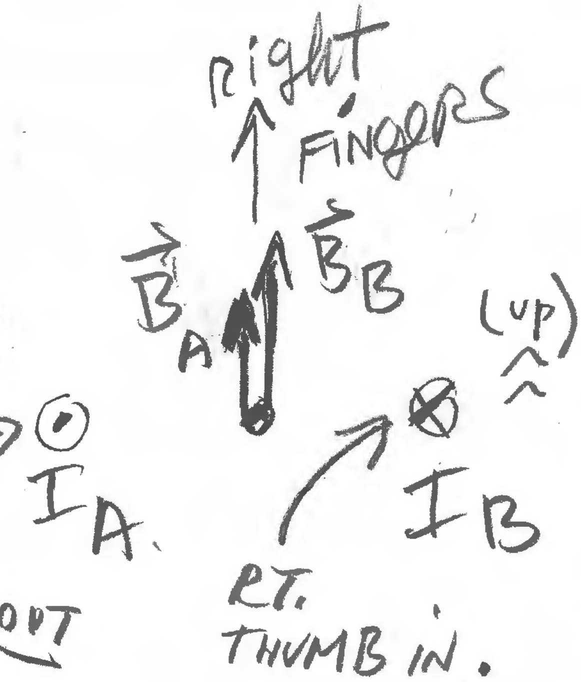
$$B = \frac{\mu_0 I}{2\pi X}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi X}, \ \underline{IN}$$

SAME RESULT
AS AMPERE'S
LAW

4.

b.



RT. THUMB OUT

a.

$$B_A + B_B = |\vec{B}_{net}|$$

$$= \frac{\mu_0}{2\pi \frac{d}{2}}\left[9.00+9.50\right]$$

$$= \frac{\mu_0}{\pi d}\left[18.50\right]$$

$$= \frac{4\pi\times10^{-7}}{\pi d}\cdot\left[18.50\right]$$

$$= \frac{4\times10^{-7}}{(0.035)}\left[18.50\right]$$

$$= (114.29)(18.50)\times10^{-7}$$

$$= 2.1\times10^{3}\times10^{-7}$$

$$= 2.1\times10^{-4}\,T = \text{MAGNITUDE}$$

$$\vec{B}_{NET} = 2.1\times10^{-4}\,T\text{, up}$$

c.

$$F = \frac{\mu_0 I_A I_B \cdot (1m)}{2\pi d}$$

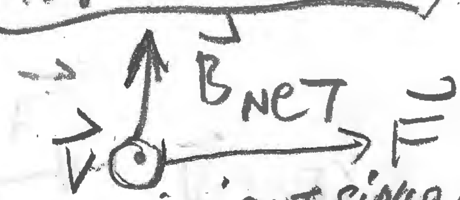
$$= \frac{4\pi\times10^{-7}(9)(9.5)(1)}{2\pi(0.035)}$$

$$= \frac{(2)(9)(9.5)}{(0.035)}\times10^{-7}$$

$$= 4.9\times10^{3}\times10^{-7}$$

$$= \boxed{4.9\times10^{-4}\ (N)}$$

d.



Force is RIGHT since $q<0$.

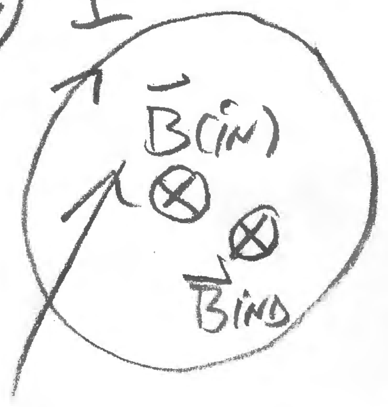
$$|\vec{F}| = F = qVB\sin 90^\circ$$

$$= qVB = 1.4\times10^{-6}\times2\times10^{6}\times(2.1\times10^{-4})$$

8

4 (a) $|\vec{F}| = 5.9 \times 10^{-4}$ (N)

5 (a.) I CW



$|\vec{B}|$ decreasing

(b.)

I

$\vec{B}_{IND}$ is IN,

from CH 28.

(c.) $|\varepsilon| = A\left|\frac{dB}{dt}\right|$

$\frac{dB}{dt} = -0.015$

$\left|\frac{dB}{dt}\right| = 0.015$

$\Rightarrow |\varepsilon| = \pi r^2 \, 0.015$

$= \pi(1)(0.015)$

$= 0.0471$ (V)

$= 47$ mV.

(d.) $I = \frac{|\varepsilon|}{800\,\Omega}$

$= \frac{0.0471}{800}$ (A)

$= 5.89 \times 10^{-5}$ A

$= 58.9\ \mu A$

9

6 copied TEXT from # 23. - CH30

CH30.

a NOTE! $\frac{\mathcal{E}}{R} = \frac{6}{8} = 0.75(A)$

$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$

$\tau = \tau_L = L/R = 0.313(s)$

$\frac{dI}{dt} = \frac{\mathcal{E}}{R}\left[-\frac{1}{\tau}e^{-t/\tau}\right]$

$= \frac{\mathcal{E}}{R} \cdot \frac{1}{L/R} e^{-t/\tau}$

$\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-t/\tau}$

at $t=0$; $\frac{dI}{dt} = \frac{\mathcal{E}}{L}$

$= \frac{6}{2.5} = 2.4 \frac{A}{S}$ at $t=0$.

b. Done in 2 steps:

A. $0.5 = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$

B. and $\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-t/\tau}$

NOW A. SAYS:

$0.5 = \frac{\mathcal{E}}{R} - \frac{\mathcal{E}}{R} e^{-t/\tau}$

$\frac{\mathcal{E}}{R} e^{-t/\tau} = \frac{\mathcal{E}}{R} - 0.5$

$0.75 e^{-t/\tau} = 0.75 - 0.5$

$0.75 e^{-t/\tau} = 0.25$

$e^{-t/\tau} = \frac{1}{3}$

$\frac{dI}{dt} = (2.4)\left(\frac{1}{3}\right)$

$= 0.8 \, A/S$ when $I = 0.5 (A)$

c. $I = \frac{\mathcal{E}}{R}\left(1 - e^{t/\tau}\right)$

(6) (c.)

NOTE: $\frac{t}{\tau}$

$= \frac{0.250(s)}{0.3}$

$= 0.799 < 1$

evaluate:

$e^{-0.799} = 0.449$

$I = 0.75(1 - 0.449)$

$= 0.413 \text{ (A)}$

(d) $I(t \to \infty)$

$= \frac{\varepsilon}{R}(1 - e^{-\infty})$

$= \frac{\varepsilon}{R} = 0.75 \text{ (A)}$