Test2pt2_4B

Follow up (Solutions below)

1. Electric positive charge Q is distributed uniformly around a *thin ring* of radius R.
(a) (30) Find the potential V at a point P at a distance x from center on the ring axis.

(b) (5) Suppose a negative charge –e with mass m starts from rest at infinity and moves toward the ring center along the axis. What is the charge's speed when it arrives at the center? Use symbols.

2. (30) Two long, coaxial cylindrical *hollow* conductors are separated by a vacuum. The inner conductor has radius *a* and linear charge density λ. The outer conductor has radius *b* and linear charge density $-\lambda$. Find the capacitance per unit length for this capacitor. Use symbols.

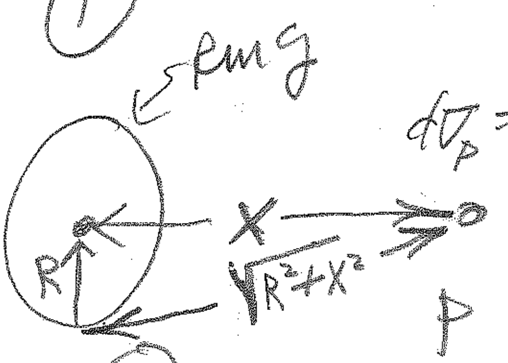
3. (35) USE SYMBOLS. A solid sphere of radius R has charge Q uniformly distributed through its volume. The energy **density** in the space surrounding the sphere is $\frac{1}{2}\varepsilon_o E^2$, where E is the electric field magnitude for r > R.

(a) (30) Use the differential volume $4\pi r^2 dr$. Find the total **energy** in Joules in the infinite space *outside* the charge.

(b) (5) Evaluate your answer to (a) in the limit R goes to zero. Explain.

Test 2 follow up solutions

(1.)



$$dV_p = \frac{k\,dq}{\sqrt{R^2+x^2}}$$

(a.)

$$V_p = \int_{\text{Ring}} dV_p = \frac{kQ}{\sqrt{R^2+x^2}}.$$

(b)

$$KE_i + U_i = KE_f + U_f$$

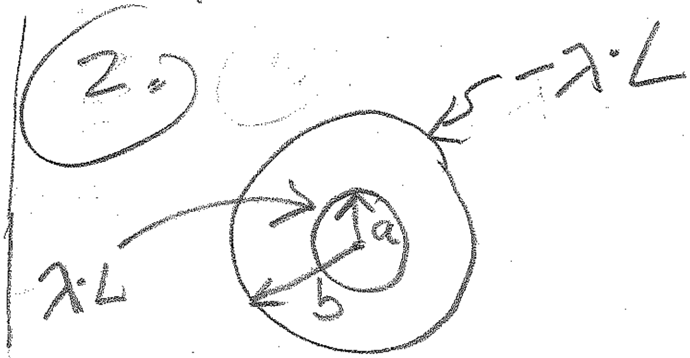
$$0 + 0 = \tfrac{1}{2}mv_f^2 + q\cdot V_f$$

$$0 = \tfrac{1}{2}mv_f^2 + (-e)\cdot\frac{kQ}{\sqrt{R^2+0^2}}$$

$$V_f = \sqrt{\frac{2ekQ}{m\cdot R}}$$

$$= \sqrt{\frac{eQ}{2\pi\varepsilon_0\cdot mR}}$$

(2.)



$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\lambda\cdot L}{V_a - V_b}$$

$$C = \frac{\lambda\cdot L}{\left|-\int_a^b E_r\cdot dr\right|}$$

$$C = \frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}},$$

since $E_r = \dfrac{\lambda}{2\pi\varepsilon_0 r}$.

and $\left|\dfrac{-\lambda}{2\pi\varepsilon_0}\ln\dfrac{b}{a}\right| = \dfrac{\lambda}{2\pi\varepsilon_0}\ln\dfrac{b}{a}$

$$C = \frac{2\pi\varepsilon_0 \cdot L}{\ln\frac{b}{a}}$$

$$\frac{C}{L} = \boxed{\frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}}$$

3. a. $Q \rightarrow$ R

$$\frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \cdot \frac{k^2Q^2}{r^4}$$

$$dU = u \cdot 4\pi r^2$$

$$dU = \frac{1}{2}\varepsilon_0 \cdot \frac{k^2Q^2}{r^2} \cdot 4\pi$$

$$U = \int_R^\infty \frac{1}{2}\varepsilon_0 \cdot \frac{k^2Q^2}{r^2} \cdot 4\pi\, dr$$

$$= 2\pi\varepsilon_0 k^2 Q^2 \int_R^\infty \frac{dr}{r}$$

$$= 2\pi\varepsilon_0 k^2 Q^2 \cdot \frac{1}{R}$$

$$= 2\pi\varepsilon_0 \cdot \frac{Q^2}{16\pi^2\varepsilon_0^2} \cdot \frac{1}{R}$$

$$\Rightarrow \boxed{U = \frac{Q^2}{8\pi\varepsilon_0 R}}$$

$$\Rightarrow \boxed{U = \frac{kQ^2}{2R}}$$

b. $U \rightarrow \infty$ as $R \rightarrow 0$.

Impossible to assemble a finite amount of charge into a single point.