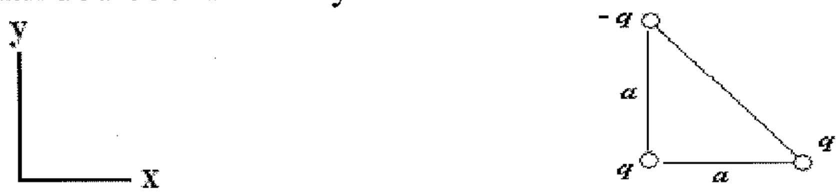
Solutions below

1. (40 points) Three point charges are arranged in an isosceles right triangle as in the figure below. There is an angle of 90 degrees between the two short sides, which have length *a*. All charges have the same magnitude $|q|$. The charge in the upper left corner has value $-q$. Most of this problem uses symbols---unless otherwise noted, *use a, q and other symbols as needed.* You will be investigating the *net force* on the charge in the *lower right corner* due to the other two charges. As needed, assume a right-handed coordinate system shown below:



(a) (8 points) What is the x- component F_{xnet} of the *net* force on the charge in the lower right corner due to the other two charges?

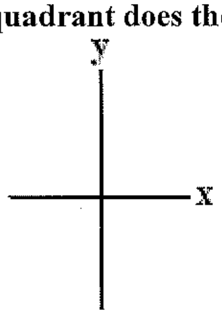
(b) (8 points) What is the y- component F_{ynet} of the *net* force on the charge in the lower right corner due to the other two charges?

(c) (8 points) What is the angle the *net* force makes with the positive x-axis? Note: In this case, your answer will be a number. Give your answer in degrees.

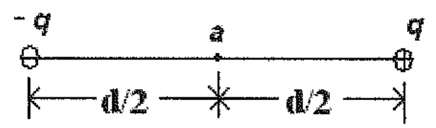
(d) (8 points) What is the *magnitude* of the net force on the charge in the lower right corner due to the other charges?

(e) (3 points) Assume $a = 1.00$ m and $q = 2.00 \times 10^{-6}$ C. Plug in these numbers to evaluate the *magnitude* of the net force on the charge in the lower right corner due to the other charges?

(f) (5 points) **Make a sketch of the net force vector on the x-y axes below with the tail of the arrow at the origin. Label the angle with the x-axis. What quadrant does the vector lie in? Your picture should indicate the correct quadrant.**



2. (40 points) ***THE ELECTRIC FIELD OF AN ELECTRIC DIPOLE.*** This is an approximate model of a water molecule. Point charges q and –q are placed a distance d apart. This combination of two charges with equal magnitude and opposite sign is called an *electric dipole.* For most of this problem use *symbols:* Use d, h, ***q*** and other symbols unless otherwise noted.



fig, 1

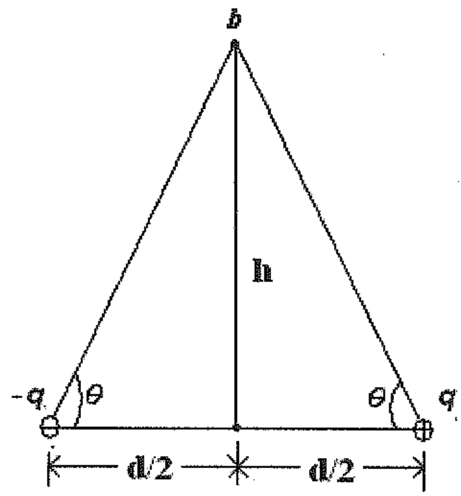
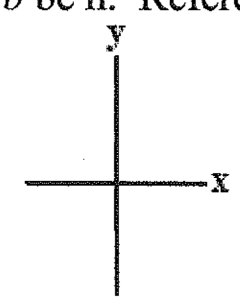


fig. 2

(a) (7 points) Refer to fig. 1 above. What must be the *direction* of the net electric field at point *a* midway between the two charges? Indicate this direction by drawing an arrow in fig. 1 with its tail at point *a*.

(b) (7 points) Refer to fig. 1 above. Symbolically, what is the *magnitude* E_{net} of the *net* electric field at point *a* midway between the two charges?

For the next parts, next page, see fig. 2. Point *b* is on the perpendicular bisector of the line connecting the two charges. Let the distance between the line connecting the two charges and point *b* be h. Reference the right handed x-y axes shown here as needed.



(c) (8 points) Use basic trigonometry to find the cosine of the angle θ shown in fig. 2 in terms of d and h.

(d) (5 points) At point *b*, what must be the y-component E_{ynet} of the *net* electric field due to the two charges? Show work.

(e) (5 points) At point *b*, what is the x-component E_{xnet} of the *net* electric field due to the two charges? Use symbols.

(f) (2 points) At point *b*, what is the direction of the *net* electric field due to the two charges? Indicate this direction by drawing an arrow in fig. 2 with its tail at point *b*.

(g) (2 points) At point *b*, what is the magnitude of the *net* electric field due to the two charges? What is the relationship between your answer here and your answer to part (e)?

(h) (4 points) Evaluate your formula found in part (g) in the limit of large h, assuming h >> d/2. Comment on the dependence of the field on an inverse power of h. What power of h is in the denominator of your formula in this limit?

3. (40 points) A *solid* insulating sphere with radius R has *total* charge *e* (in coulombs C) *uniformly* distributed through out its volume. (By definition, the insulating sphere is NOT a conductor!) For parts (a) through (d), see figure 1 on the next page. (e) and (f) use fig. 2.

(a) (4) What is the charge density of the sphere? Your answer should only depend on *e*, R and other symbols.

(b) (8) What is the magnitude of the electric field E *outside* the sphere at a distance 2R from the center? Your answer will only contain *e*, R and other symbols.

(c) (8) What is the magnitude of the electric field E *inside* the sphere at a distance R/2 from the center? Your answer will only contain *e*, R and other symbols.

(d) (8) Sketch E as a function of r on the axis shown below. r is the distance from the center.

(e) (8) Now refer to fig. 2 on the next page. Suppose a point particle of negative charge –e charge is placed a distance r from the center as shown, where r < R. What is the direction of the net force on this point charge? Indicate the direction by drawing an arrow with tail aligned near the point charge. What is the magnitude of the net force on the point charge? Use *e*, R, r and other symbols.

(f) (4) Refer to figure 2. If the point charge starts from rest at the position shown, what is the angular frequency ω of oscillations about r = 0? Assume the point charge has mass *m*. Your answer will be in terms of *m, R, e* and other symbols. Show work.

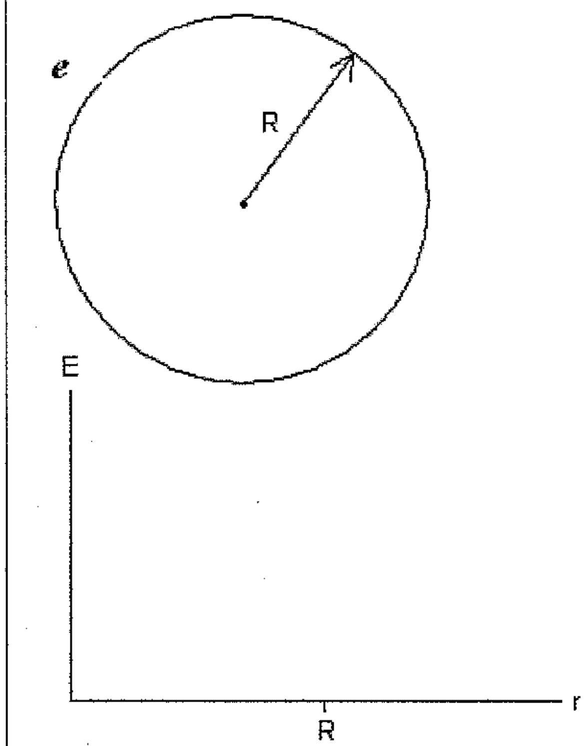


fig.1

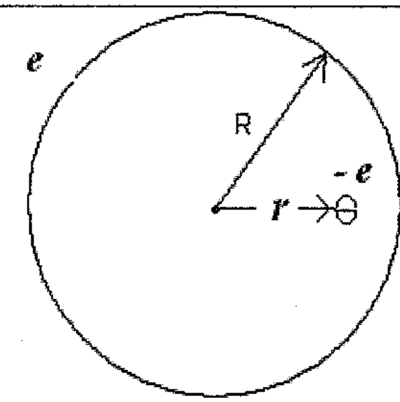
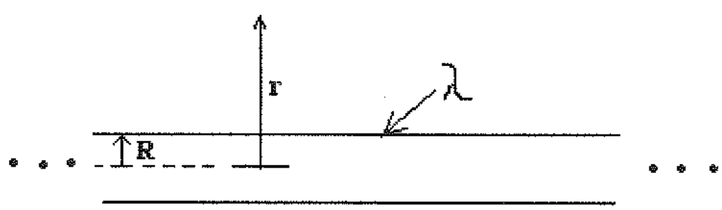


fig. 2

4. (30 points) ELECTRIC FIELD DUE TO AN INFINITE *CONDUCTING* WIRE IN ELECTROSTATIC EQUILIBRIUM.

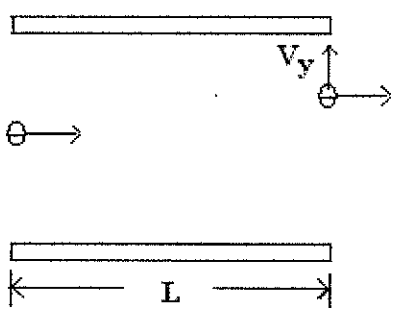
Below is shown a section of a solid, straight *conducting* wire, of *infinite length*, with linear charge density λ. Assume λ >0. λ gives the charge per unit length along the wire even though the charge is on the *surface* of the wire in three dimensions. In other words, λ gives the amount of that 3D charge per meter of wire length in C/m.

The wire radius is R. The *variable* r is the distance from the wire's central axis as shown. Your answers below may be written in terms of λ, r and other symbols as needed.



(a) (8 points) What is the magnitude of the electric field E for $r < R$. Explain.
(b) (14 points) What is the magnitude of the electric field E for $r \geq R$. For *full credit*, you must *derive* your answer using Gauss's Law and the appropriate Gaussian surface.
(c) (2 points) For $r \geq R$, what is the angle between the electric field vector and the wire's surface?
(d) (6 points) How a Geiger counter works: Suppose an electron of mass m and *negative* charge $-e$ is a distance r from the wire's center, assuming $r > R$. Using symbols, find the magnitude of the particle's *acceleration*. *Ignore* the gravitational force. What is the direction of the acceleration, *toward* or *away* from the wire? Explain.

5. (16 points) An electron moves initially to the right as it enters a region between two, thin parallel plates that have surface charges of equal magnitude and opposite sign. When the electron enters the region between these two charged plates, the *initial* horizontally directed speed is 6.00×10^{6} m/s as shown by the rightward arrow at the left end of the plates. The uniform electric field between the plates has magnitude E = 12.00 N/C. The time the electron spends between the plates is t = 0.500 μs. When the electron finally exits the region between the plates, the electron has picked up a y-component of velocity V_y shown by the upward vertical arrow at the right end of the plates. Let the positive y-direction be upward and the positive x-direction be rightward.



(a) (2 points) Which plate is positive, the bottom or the top one? Explain.
(b) (2 points) What is the direction of the electric field between the plates up or down? Explain.
(c) (4 points) What is the magnitude of the *surface* charge density σ (in C/m^2) on either plate? For full credit, explain any formula you use to get the answer.
(d) (4 points) What is the value of V_y ? The electron mass is $m = 9.11 \times 10^{-31}$ kg and the charge is -1.60×10^{-19} C.
(e) (4 points) What is the length L of the plates?

Sp'14 Test 1 solutions P4B!

1.

a.

$$F_{x_{net}} = \frac{kq^2}{a^2} - \frac{kq^2}{(\sqrt{2}a)^2}\frac{\sqrt{2}}{2}$$

$$= \frac{kq^2}{a^2} - \frac{kq^2}{2a^2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{kq^2}{a^2}\left(1 - \frac{\sqrt{2}}{4}\right) > 0$$

b.

$$F_{net_y} = 0 + \frac{kq^2}{(\sqrt{2}a)^2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{kq^2}{4a^2}\sqrt{2}$$

$$= \frac{kq^2}{a^2}\cdot\frac{\sqrt{2}}{4} > 0$$

c.

$$\tan\theta_r = \frac{|F_{net\,y}|}{|F_{net\,x}|}$$

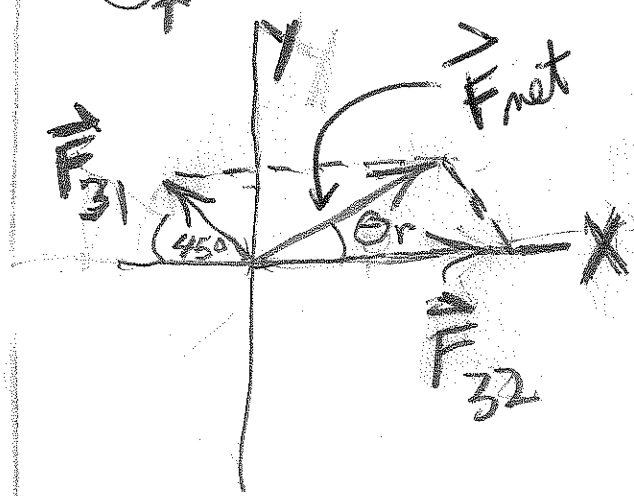
$$= \frac{\frac{kq^2}{a^2}\cdot\frac{\sqrt{2}}{4}}{\frac{kq^2}{a^2}\left(1-\frac{\sqrt{2}}{4}\right)}$$

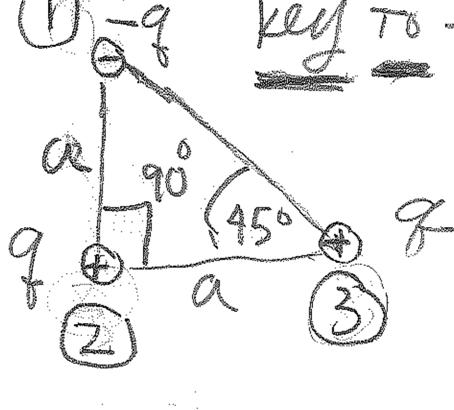
$$= \frac{\sqrt{2}}{4-\sqrt{2}}$$

$$= 0.5469$$

$$\rightarrow \tan\theta_r = 0.5469$$

$$\theta_r = 28.675^\circ$$





key to force notation.

(d.)

$$F_{net} = \sqrt{F_{net_x}^2 + F_{net_y}^2}$$

$$= \frac{kq^2}{a^2} \cdot \sqrt{\left(1-\frac{\sqrt{2}}{4}\right)^2 + \left(\frac{\sqrt{2}}{4}\right)^2}$$

(e.)

$$= \frac{kq^2}{a^2} \cdot \sqrt{(0.6464)^2 + (0.3536)^2}$$

$$= 0.7368 \cdot \frac{kq^2}{a^2}$$

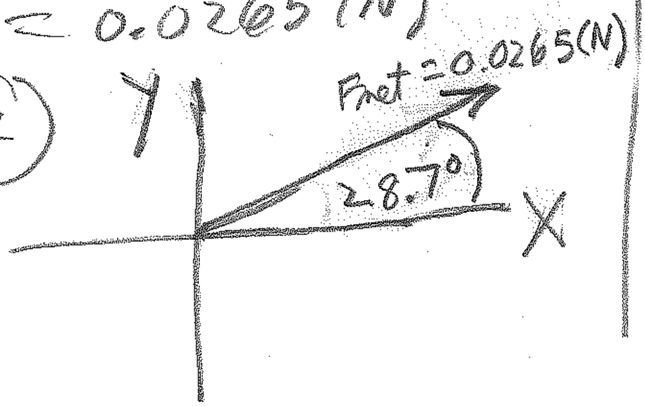
$$= 0.7368 \cdot \frac{(9\times10^9)(2\times10^{-6})^2}{(1)^2}$$

$$= 26.526\times10^{-3} \ (N)$$

$$= 2.65\times10^{-2} \ (N)$$

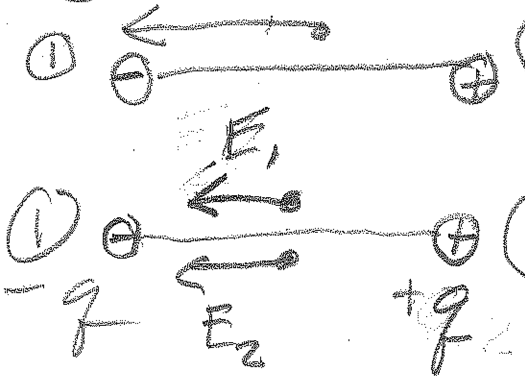
$$= 0.0265 \ (N)$$

(f)



(2)

(a.) $E_{net} = E_1 + E_2$

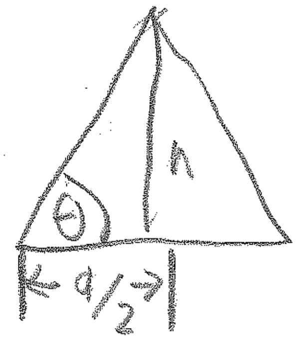


(b.) $E_{net} = 2E$

$E_1 = E_2 = E$

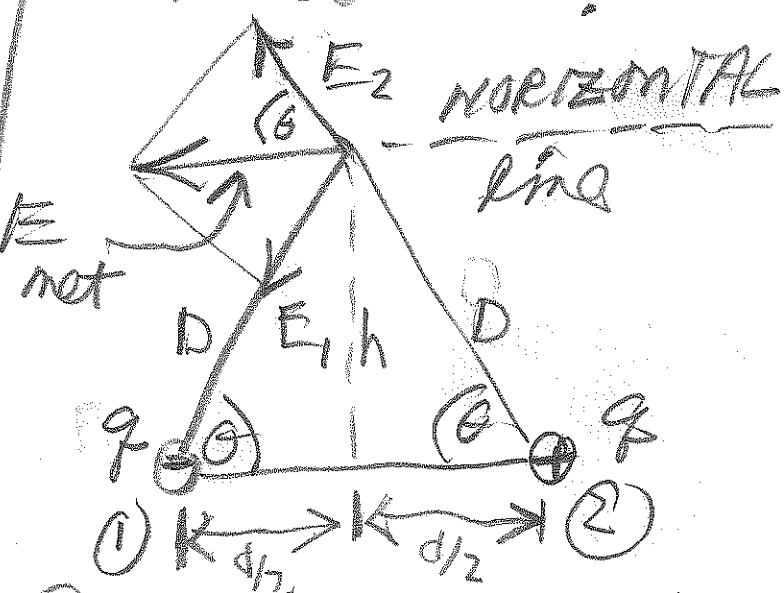
$$E_{net} = \frac{2kq}{(d/2)^2} = \frac{8kq}{d^2}$$

(c.)



(c.) $$\cos\theta = \frac{(d/2)}{\sqrt{h^2 + \frac{d^2}{4}}}$$

(d.) O - use symmetry and parallelograms rule:



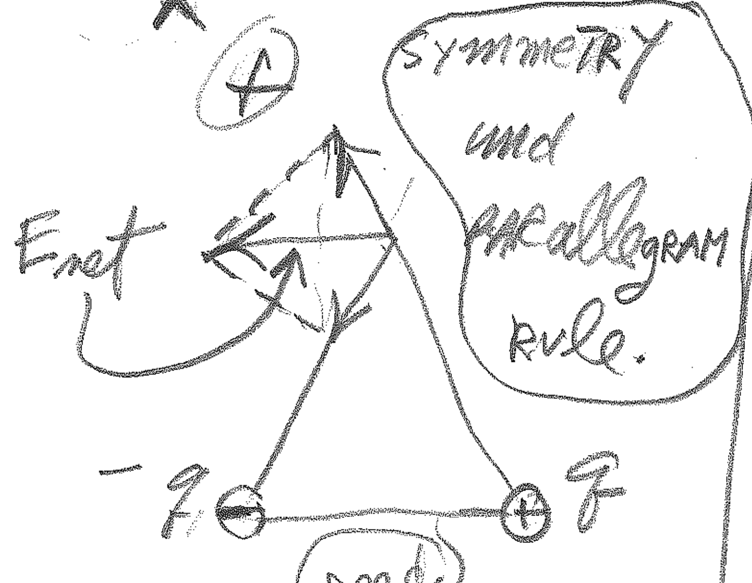
(e.) $$E_{net\,x} = -2 \cdot E \cos\theta$$

$$= -2 \cdot \frac{kq}{\left(h^2 + \frac{d^2}{4}\right)} \cdot \frac{\frac{d}{2}}{\left(h^2 + \frac{d^2}{4}\right)^{\frac{1}{2}}}$$

2. e.

$$E_{net\,x} = \frac{-kqd}{(h^2 + d^2/4)^{3/2}}$$

f.



g) $E_{net} = |E_{net\,x}|$

$$= \frac{kqd}{(h^2 + \frac{d^2}{4})^{3/2}}$$

h. let $h >> \frac{d}{2}$.

THUS

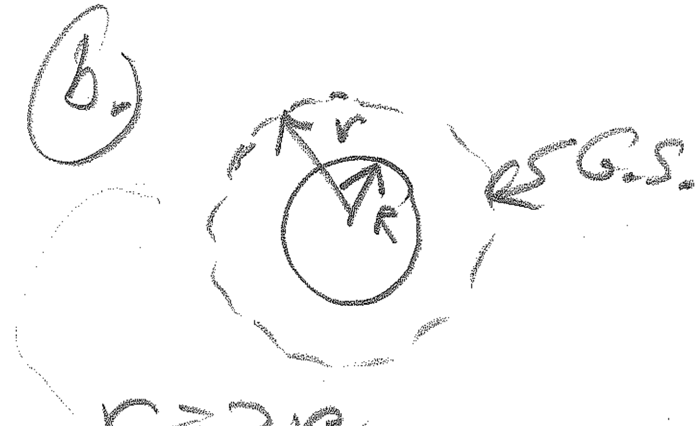
$$E_{net} \to \frac{kqd}{(h^2)^{3/2}}$$

$$E_{net} = \frac{kqd}{h^3}$$

3. a

$$\rho = \frac{e}{\frac{4}{3}\pi R^3} = \frac{3e}{4\pi R^3}$$

b.



$r = 2R$

$$E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{e}{\varepsilon_0}$$

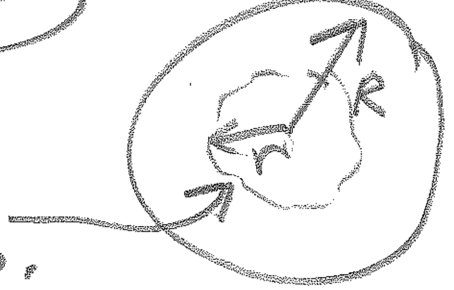
$$E = \frac{e}{4\pi\varepsilon_0 r^3}$$

$R = 2R$!

$$E = \frac{e}{16\pi\varepsilon_0 R^2}$$

$$= \frac{ke}{4R^2}$$

(3.) (c.)



G.S.

$$E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0}$$

$Q_{encl} = \rho \cdot \text{volume}$

volume = G.S. volume

$\text{volume} = \frac{4}{3}\pi r^3$

$$Q_{encl} = \rho \cdot \frac{4}{3}\pi r^3$$

$$= \frac{3e}{4\pi R^3} \cdot \frac{4}{3}\pi r^3$$

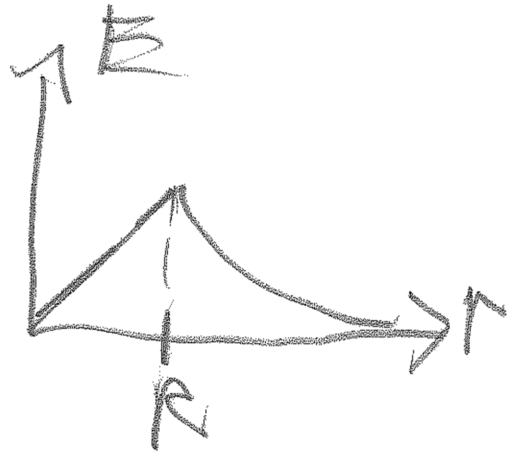
$$= \frac{er^3}{R^3}$$

$$E(4\pi r^2) = \frac{er^3}{\varepsilon_0 R^3}$$

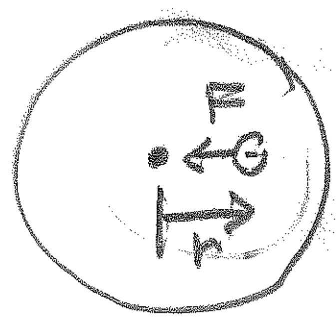
(c) $E = \dfrac{er}{4\pi\varepsilon_0 R^3}$

$$E = \frac{ker}{R^3} = \frac{ke(R/2)}{R^3} = \frac{ke}{2R^2}$$

(d)



(e)



$F = eE$

$$F = \frac{ke^2 r}{R^3}$$

(f) $ma_r = -\dfrac{ke^2 r}{R^3}$

(3.)

(4.)

$$m\frac{d^2r}{dt^2} = -\frac{ke^2 r}{R^3}$$

$$r = A\cos \omega t$$

$$\omega = \sqrt{\frac{ke^2/R^3}{m}}$$

$$\omega = \sqrt{\frac{ke^2}{mR^3}}$$

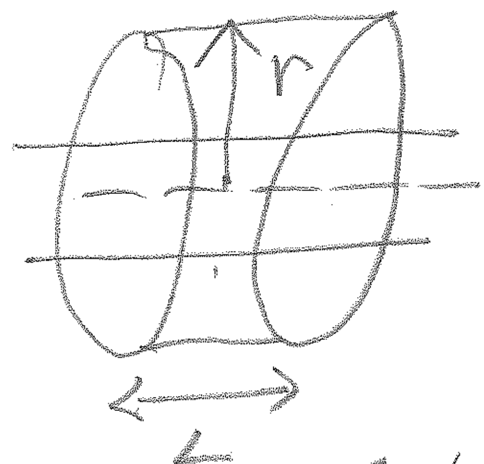
$$k = \frac{1}{4\pi\varepsilon_0}$$

$$\omega = \sqrt{\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{mR^3}}$$

(4) (a) $E = 0$, $r < R$

inside a conductor

(b) $r > R$



$$E(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$

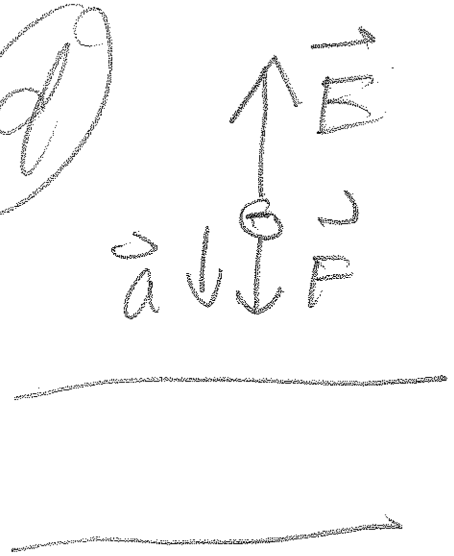
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

(c.) 90°

$\vec{E} \perp$ SURFACE

(4.)

(d)



$\vec{F} = -e\vec{E} = m\vec{a}$.

$-e < 0$.

$\Rightarrow \vec{F} = -e \cdot \vec{E}$,

NOTE: $\vec{F}$ and $\vec{a}$ ARE TOWARD

WIRE:

$|\vec{E}| = \frac{e \cdot \lambda}{2\pi\varepsilon_0 r} = |m\vec{a}|$

$|\vec{a}| = \frac{e\lambda}{2\pi\varepsilon_0 m r}$

(5.)

(a.) top; negative

electron attracted

to positive plate

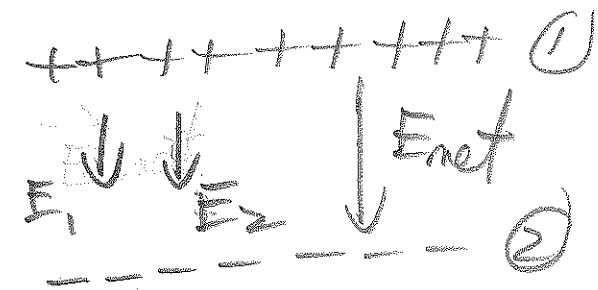
(b.) DOWN, $\vec{E}$,

points from +ve

charge to −ve

charge

(c.)



Between plates

$E_{net} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$

$E_{net} = 12.00 \frac{N}{C} = \frac{\sigma}{\varepsilon_0}$

$\sigma = (8.85 \times 10^{-12})(12.00) \frac{C}{m^2}$

$\sigma = 1.06 \times 10^{-12} \frac{C}{m^2} = 1.06 \times 10^{-10} \frac{C}{m^2}$

5. d.

$$V_y = a_y \cdot t$$

$$= \frac{-eE_y}{m} \cdot t$$

$$= \frac{(-1.6\times10^{-19})(-12.00)(0.500\times10^{-6})}{9.11\times10^{-31}}$$

$$= +1.05\times10^{-25+31}$$

$$= +1.05\times10^{6}\ \frac{m}{s}$$

e. $V_x \cdot t = L$

$$= (6.00\times10^{8})(0.500\times10^{-6})\ (m)$$

$$= 3.00\ m.$$