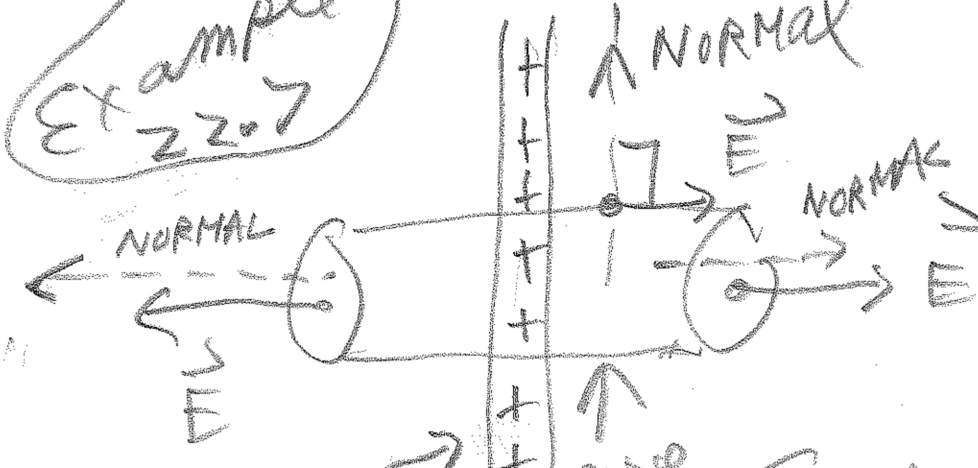
Supplementary notes:

Gauss's Law

$$|\vec{dA}| = dA$$

$$\vec{dA} \perp \text{surface}$$

Example Ex 22.7



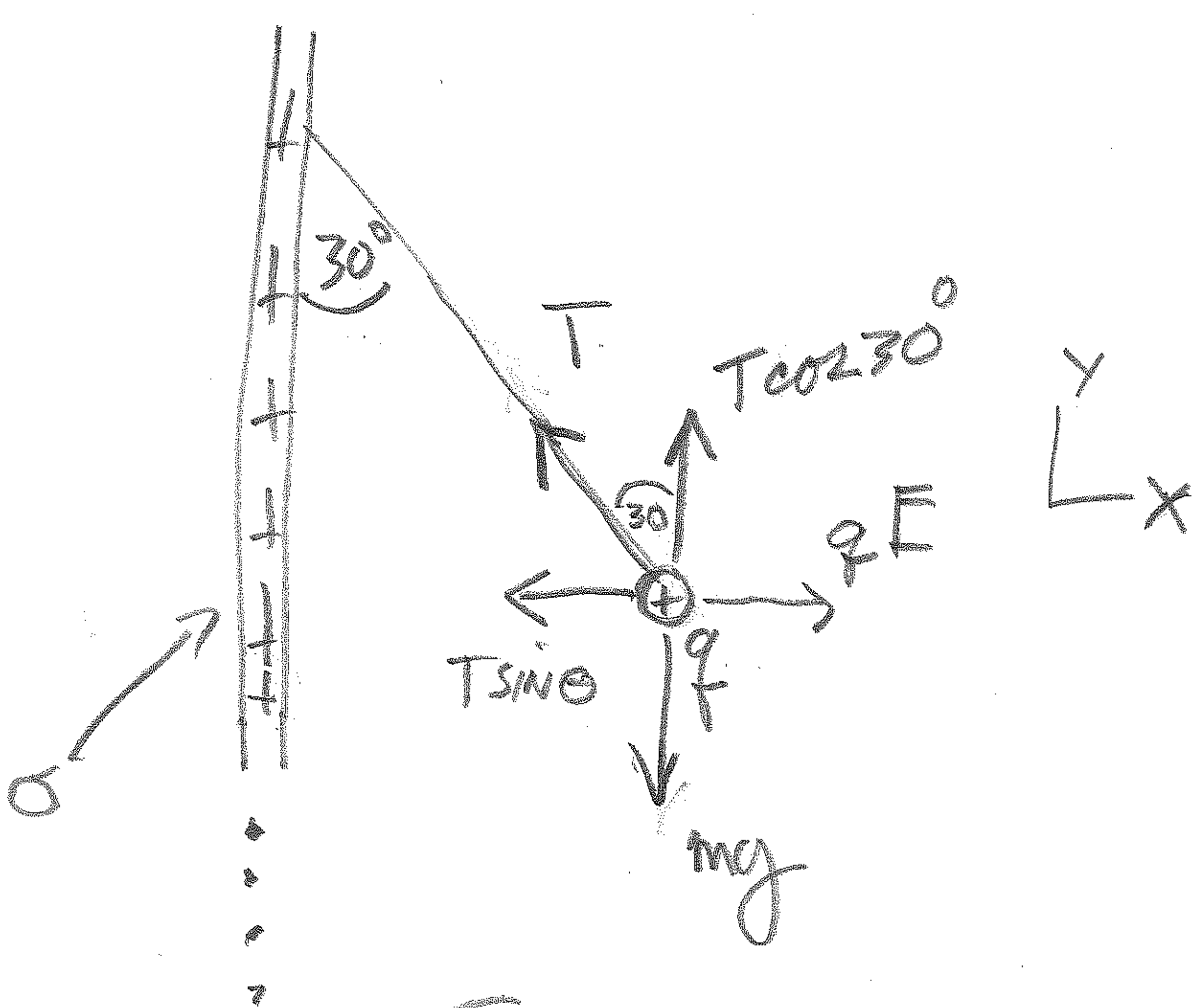
$$Q_{enc} = \sigma \cdot A$$

$$\oint E\,dA\cos\theta = \frac{Q_{enc}}{\varepsilon_0}$$

$$\underbrace{\int E\,dA\cos\theta}_{\text{End caps}} + \underbrace{\int E\,dA\overbrace{\cos\theta}^{\to 0}}_{\text{curved side}}$$

$\cos 90^\circ$

SAMPLE T2 #2 8



$$\sum F_x = 0 = \text{pos} - \text{neg}$$

$$0 = \text{pos} - \text{neg}$$

$$0 = qE - T\sin\theta \Rightarrow T\sin\theta = qE \quad (1^*)$$

NOTE: YOU MUST DERIVE

$$E = \frac{\sigma}{2\varepsilon_0} \text{ (GAUSS'S LAW)}$$

DERIVE!

Sample T2 #2

$$\Sigma F_y = pos - neg$$

$$0 = T\cos 30 - mg$$

$$T\cos 30 = mg \quad 2$$

$$\frac{1}{2} \Rightarrow \tan\theta = \frac{qE}{mg}$$

$$\Rightarrow E = \frac{mg\tan 30^\circ}{q}$$

$$\frac{\sigma}{2\varepsilon_0} = \frac{mg\tan 30^\circ}{q}$$

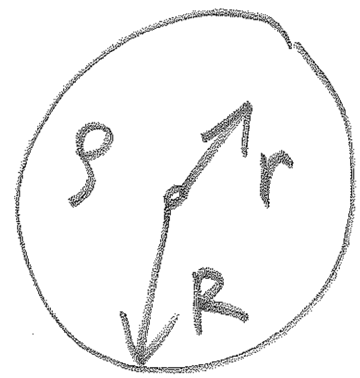
$$\Rightarrow \sigma = \frac{2\varepsilon_0\, mg\tan 30}{q}$$

$$\sigma = \frac{2(8.85\times10^{-12})(1\times10^{-3})(0.5774)}{2\times10^{-8}}\ \frac{C}{m^2}$$

$$= 5.11\times10^{-12}\cdot10^{-3}\cdot10^{8}$$

$$= 5.11\times10^{-7}\ \frac{C}{m^2}$$

Sample T2 #3



ρ = charge density $\left(\frac{C}{m^3}\right)$

$$\rho = \rho_0 \left(\frac{r}{R}\right)^3$$

(a) $E(r)$, $r \leq R$

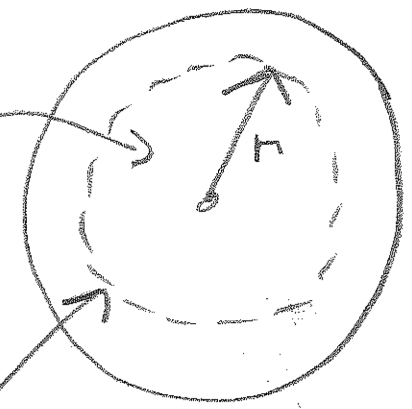
(b) $E(r)$, $r > R$

PREREQUISITE IS EXAMPLE <u>22.9</u>

see "Appendix" at end of the <u>notes</u>.

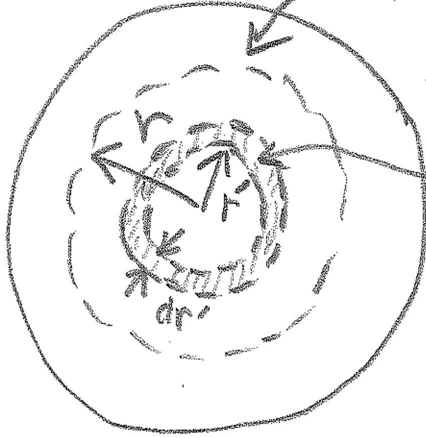
Sample test 2, #3

$q_{ENC} = Q_{encl}$



G.S.

G.S.



$dq = \rho \cdot dV$

$$dV = 4\pi r'^2 \cdot dr'$$

$$dq = \rho(r') \cdot 4\pi r'^2 dr'$$

$$Q_{encl} = \int_0^r \rho_0 \left(\frac{r'}{R}\right)^3 \cdot 4\pi r'^2 dr' = \frac{\rho_0}{R^3} \cdot 4\pi \int_0^r r'^5 dr'$$

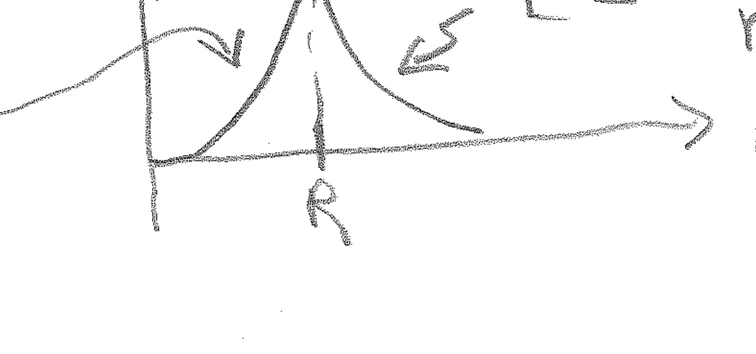
$$Q_{encl} = \frac{\rho_0}{R^3} \cdot 4\pi \frac{r^6}{6}$$

$$\underbrace{E \cdot (4\pi r^2)}_{\oint E\,dA\cos 0} = \frac{\rho_0}{R^3} \cdot \frac{4\pi}{\varepsilon_0} \cdot \frac{r^6}{6} = \frac{Q_{encl}}{\varepsilon_0}$$

G.S.

$$E = \frac{1}{4\pi r^2} \cdot \frac{\rho_0}{R^3} \cdot \frac{4\pi}{\varepsilon_0} \cdot \frac{r^6}{6}$$

$$\boxed{E = \rho_0 \frac{r^4}{6\varepsilon_0 R^3}}, \quad r \le R$$



$$E = \frac{KQ}{r^2}$$

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now evaluate E for $r > R$

$$E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0}$$

But Q_{encl} = TOTAL CHARGE Q.

$$Q = \int_0^R \frac{\rho_0}{R^3} \cdot 4\pi r'^5 dr'$$

$$= \frac{\rho_0}{R^3} \cdot 4\pi \frac{R^6}{6} = \frac{2}{3}\pi \rho_0 \cdot R^3$$

$$E(4\pi r^2) = \frac{2}{3} \cdot \frac{\pi}{\varepsilon_0} \cdot \rho_0 R^3$$

$$E = \frac{1}{4\pi r^2} \cdot \frac{2}{3} \cdot \frac{\pi}{\varepsilon_0} \cdot \rho_0 R^3$$

$$E = \rho_0 \cdot \frac{R^3}{6\varepsilon_0 r^2}$$

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NOTE, FOR $r > R$ CHECK TOTAL CHARGE Q:

$$E = \rho_0 \frac{R^3}{6\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

$$\rho_0 \frac{R^3}{6\varepsilon_0 r^2} = \frac{kQ}{r^2}$$

$$\Rightarrow \frac{Q}{4\pi\varepsilon_0} = \frac{\rho_0 \cdot R^3}{6\varepsilon_0}$$

$$\rightarrow Q = 4\pi\varepsilon_0 \cdot \frac{\rho_0 R^3}{6\varepsilon_0}$$

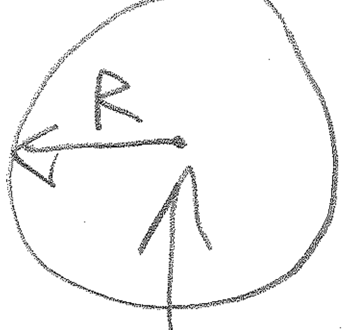
$$Q = \frac{2}{3} \cdot \pi \rho_0 \cdot R^3$$ AS before.

see #58, ch 22

see ALSO # 65.

QUIZ 2 HINTS:

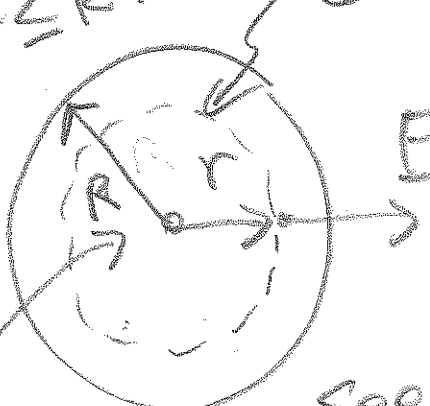
54. see Example 22.9



uniformly charged sphere with TOTAL CHARGE $Q = +e$.

FOR $r \leq R$:

G.S.



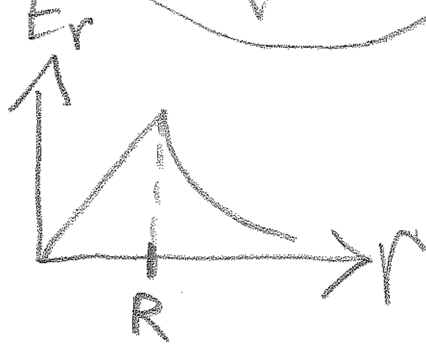
$Q_{enc} \to 0$

see Example 22.9

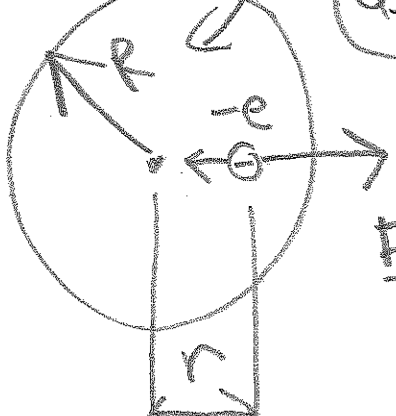
$$E_r = \frac{Q}{4\pi\varepsilon_0 R^3} \cdot r$$

WHERE $Q = +e$.

$$\left[r \leq R \right]$$



$Q_{TOTAL} = +e$



(b.) $F_r = ma_r$

$$ma_r = -e \cdot E_r$$

$$ma_r = -e \cdot \left(\frac{+e \cdot r}{4\pi\varepsilon_0 R^3} \right)$$

$$ma_r = -\frac{e^2 r}{4\pi\varepsilon_0 R^3}$$

NOTE: $a_r = \frac{d^2r}{dt^2}$

THUS,

$$m \cdot \frac{d^2r}{dt^2} = -\frac{e^2 r}{4\pi\varepsilon_0 R^3}$$

review:

CH. 14

spring and mass

$$m \cdot \frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$x = A\cos(\omega t + \phi)$$

THUS:

$$\omega = \sqrt{\frac{(e^2/4\pi\varepsilon_0 R^3)}{m}}$$

$$\boxed{r = A\cos(\omega t + \phi)}$$

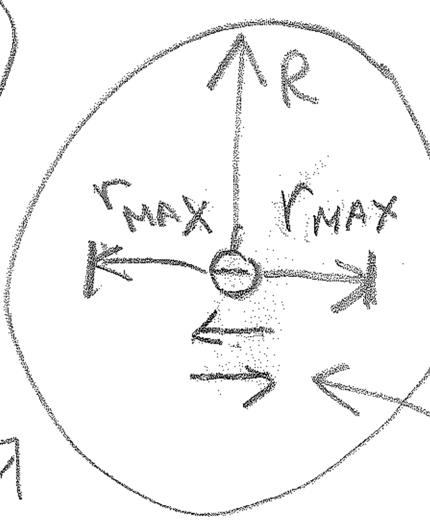
$$\omega = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m R^3}} = e \cdot \sqrt{\frac{1}{4\pi\varepsilon_0 m R^3}}$$

NOTE:

$$\boxed{\omega = 2\pi f} \qquad \left(\frac{RAD}{s} = \frac{1}{s} = s^{-1}\right)$$

f units $= Hz = s^{-1}$
$= \frac{cycles}{s}$

electron oscillates about $r=0$



oscillation frequency $= f$ (Hz)

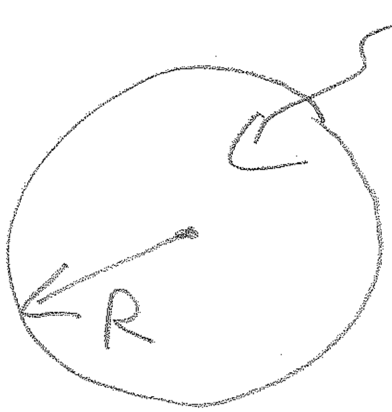
$$r = r_{MAX} \cdot \cos(\omega t + \phi)$$

$$= r_{MAX} \cdot \cos(2\pi f \cdot t + \phi)$$

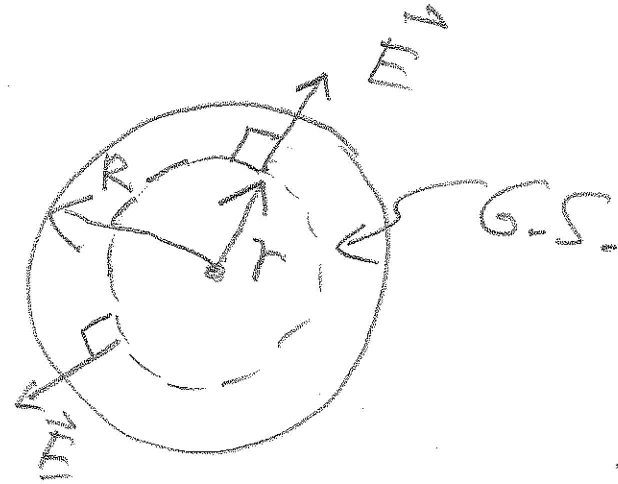
$$T = \text{period} = \frac{1}{f} \text{ (sec)}$$

$$\rightarrow \boxed{f = \frac{\omega}{2\pi}}$$

Appendix: Example 22.9



FIND E:



(a) $r < R$

$$\oint E\,dA\cos\theta = \frac{Q_{encl}}{\varepsilon_0}$$

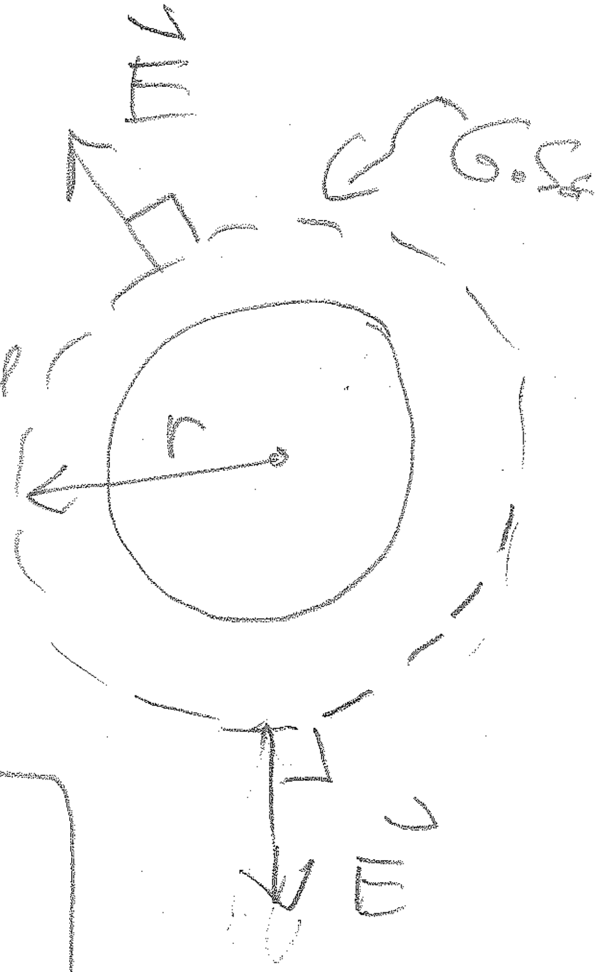
$$E(4\pi r^2) = \frac{Q_{encl}}{\varepsilon_0}; \quad Q_{encl} = \rho \cdot \frac{4}{3}\pi r^3$$

where $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3} \Rightarrow Q_{encl} = Q \cdot \left(\dfrac{r^3}{R^3}\right)$

$$\Rightarrow E(4\pi r^2) = \frac{Q}{\varepsilon_0} \cdot \frac{r^3}{R^3} \Rightarrow \boxed{E = \frac{Qr}{4\pi\varepsilon_0 R^3}}$$

Example 22.9

(b.) $r \geq R$



$$\oint E\,dA\cos 0 = \frac{Q_{encl}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \boxed{\frac{Q}{4\pi\varepsilon_0 r^2} = \frac{KQ}{r^2}}$$

SUMMARY: