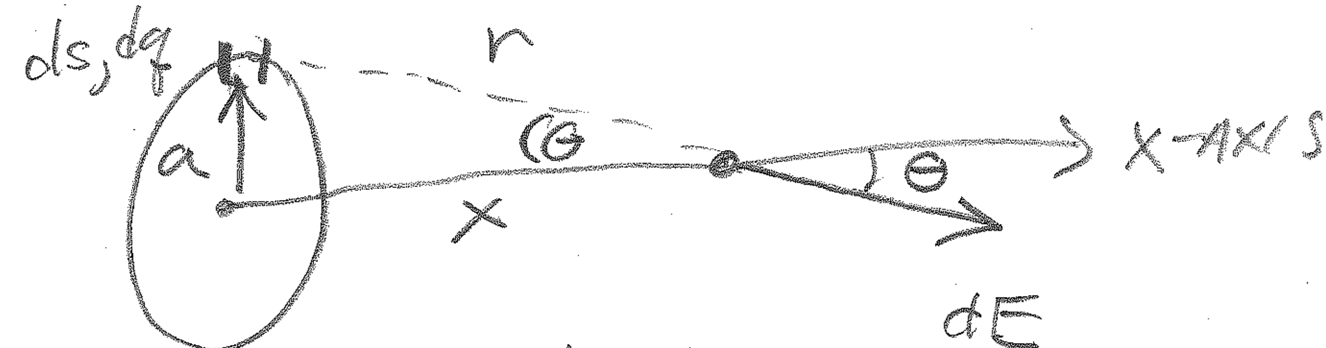
4B 1-27-14 ~~lecture~~ 1

Sec. 21.5 $\vec{E}$ - field computations

PREREQUISITE: a sound foundation in Math I, ≥ differential and integral calculus.

Examples

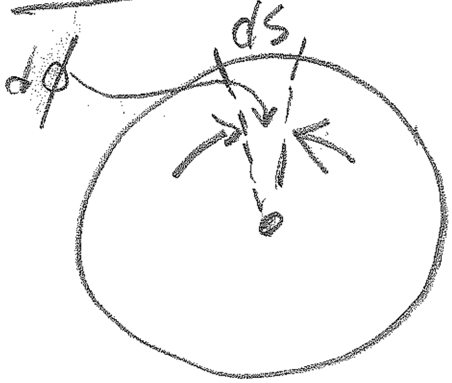
(21.9) Ring of charge



$$dE = \frac{k\,dq}{r^2} = \frac{k\,dq}{(x^2+a^2)}$$

from Coulomb's LAW. (Sec. 21.3) (Sec. 21.4) negative

top view (LOOKING IN $-x$-DIRECTION



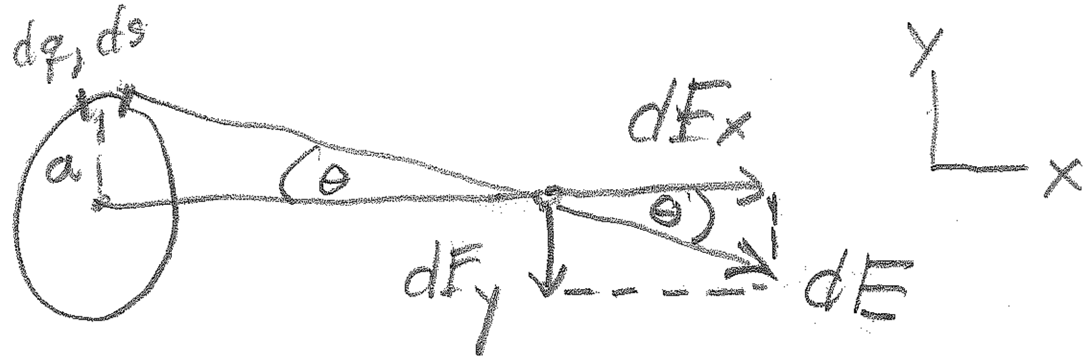
$ds = a\,d\phi$

$dq = \lambda \cdot ds,$

where λ = linear charge density (assumed constant) $\left(\frac{C}{m}\right)$

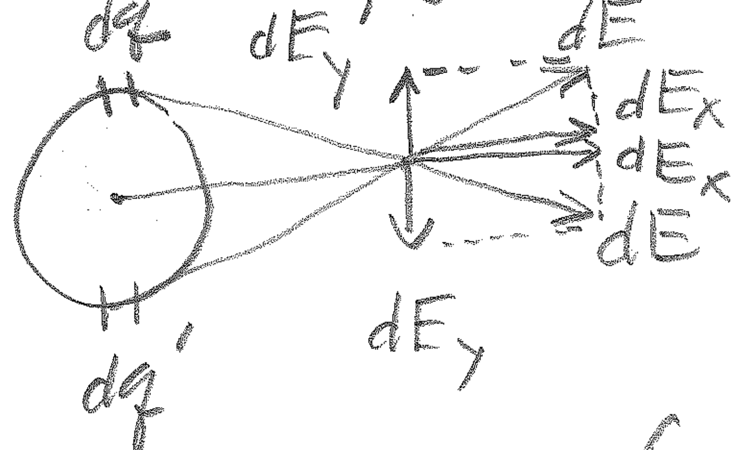
$$\Longrightarrow dE = \frac{k \cdot \lambda \cdot a\,d\phi}{(x^2+a^2)}.$$

Note: x, a are constant during integral.



<u>NOTE</u>: $\int_{ring} dE_y = 0$

Reason:



$$dE_y + dE_y' = 0$$

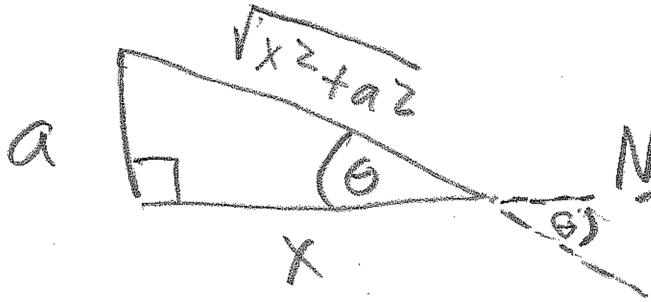
$\Rightarrow$ PAIRS cancel AROUND RING.

$$dE_x = dE \cdot \cos\theta$$ NOTE:

THUS:

$$\int_{\text{RING}} dE_x = E_{net_x} = \int_{\text{ring}} \frac{k\cdot\lambda\cdot a\,d\phi}{(x^2+a^2)}\cdot\cos\theta$$

NOTE: $$\cos\theta = \frac{x}{\sqrt{x^2+a^2}}$$

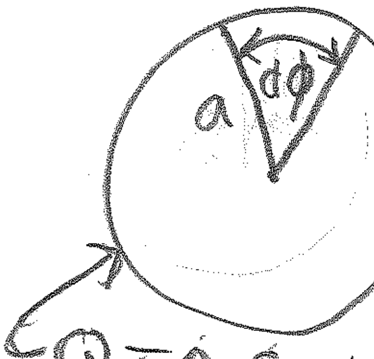


NOTE: $$\sqrt{x^2+a^2} = (x^2+a^2)^{\frac{1}{2}}$$

THUS:

$$dE_{net_x} = \frac{k\cdot\lambda\cdot a\cdot x}{(x^2+a^2)^{3/2}}\cdot\int_0^{2\pi} d\phi$$

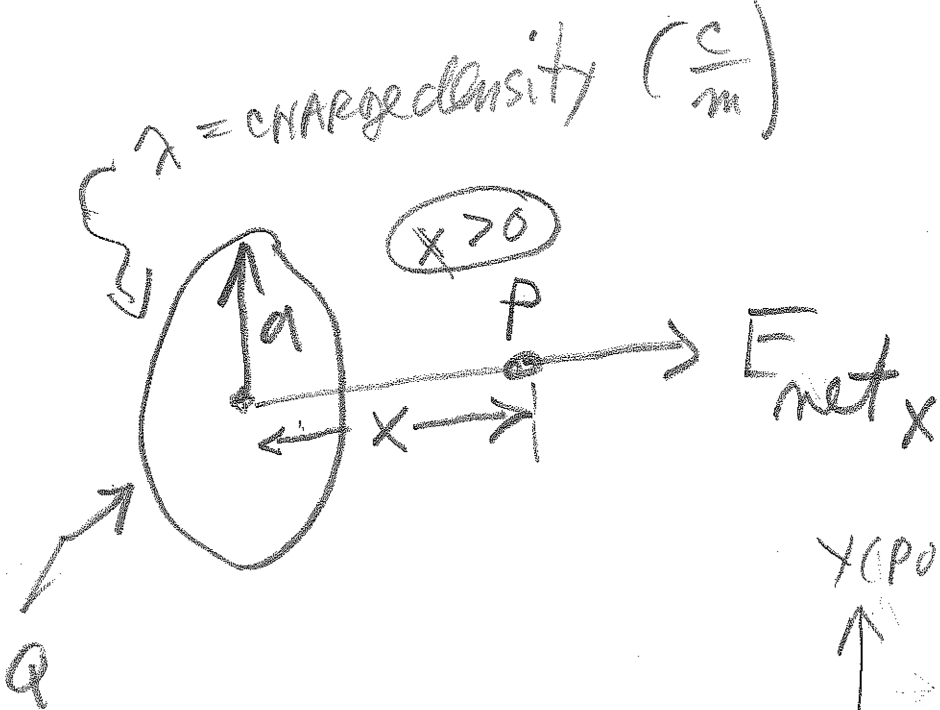
$$= \frac{k\cdot\lambda\cdot a\cdot x}{(x^2+a^2)^{3/2}}\cdot 2\pi$$

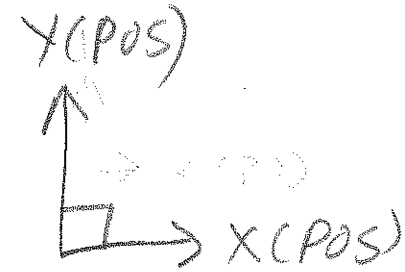


$$= \frac{k\,\lambda\cdot(2\pi a)\cdot x}{(x^2+a^2)^{3/2}} = \frac{k\cdot Q\cdot x}{(x^2+a^2)^{3/2}}$$

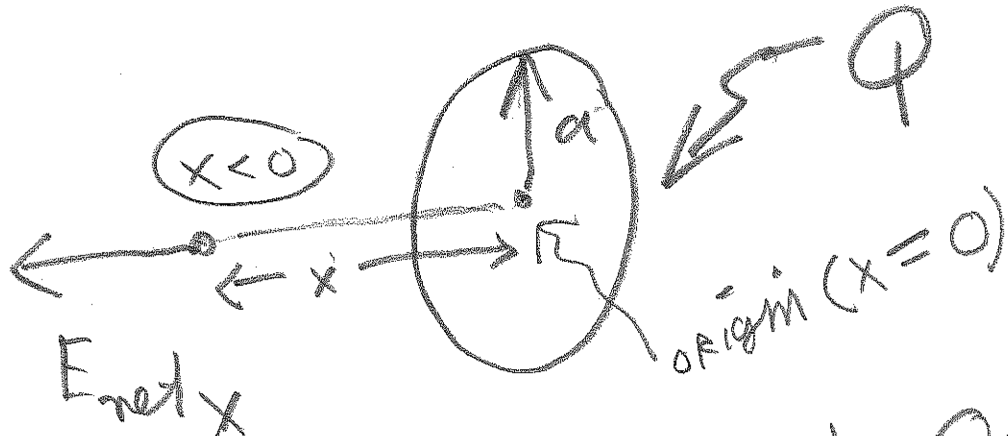
$Q = \lambda\cdot\int a\cdot d\phi = \lambda\cdot 2\pi a$

NOTE: $\lambda\cdot(2\pi a) = Q$ = TOTAL CHARGE

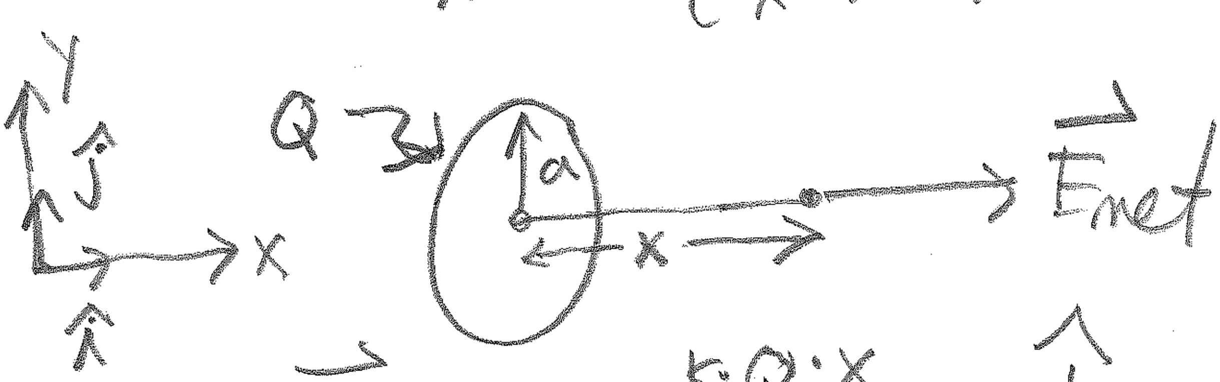




OR

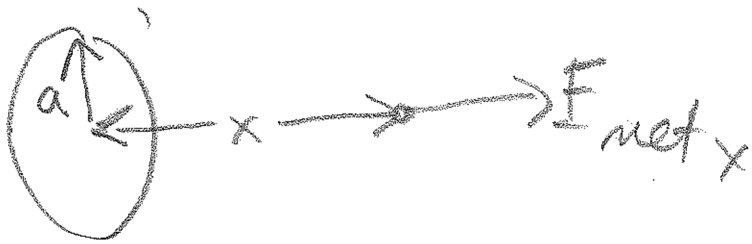


$$E_{net_x} = \frac{K \cdot Q \cdot X}{(x^2 + a^2)^{3/2}}$$



$$\vec{E}_{net} = \frac{K \cdot Q \cdot x}{(x^2 + a^2)^{3/2}} \cdot \hat{\imath}$$

(5)



$$E_{net_x} = \frac{kQx}{(x^2+a^2)^{3/2}}$$

<u>AT HOME:</u>

THINK about 2 limits

(i) $x \gg a$ (RING LOOKS LIKE a point CHARGE)

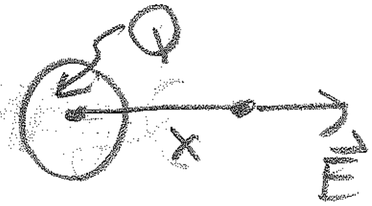
(ii) $x = 0$ ($\vec{E}_{net} = 0$)

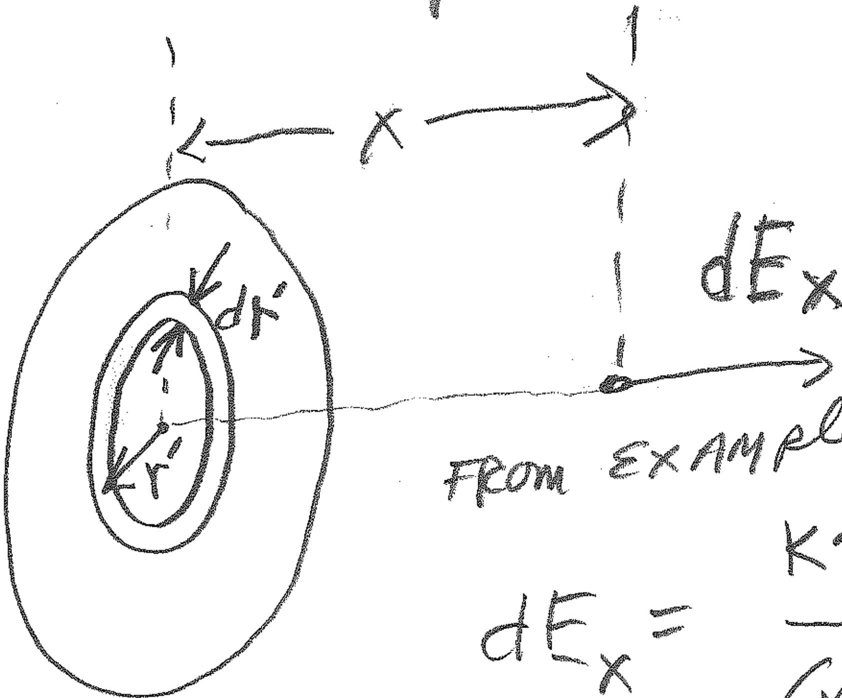
(i) $$E_{net_x} = \lim_{x \gg a} \frac{kQx}{(x^2+a^2)^{3/2}} = \frac{kQx}{(x^2)^{3/2}} = \frac{kQx}{x^3}$$

$$E_{net_x} = \boxed{\frac{kQ}{x^2}}$$

(ii) $$E_{Net_x} = \lim_{x \to 0} \frac{kQx}{(x^2+a^2)^{3/2}} = \frac{k \cdot Q \cdot 0}{(a^2)^{3/2}} = 0$$

Example 21.11 DISK of CHARGE:





FROM EXAMPLE 21.9:

$$dE_x = \frac{k \cdot dq' \cdot x}{(x^2 + r'^2)^{3/2}}$$

WHERE $dq' = \sigma \cdot 2\pi r' dr'$,

where σ = areal CHARGE density $\left(\frac{C}{m^2}\right)$.

$$E_x = k \cdot x \cdot \int_{\text{DISK}} \frac{dq'}{(x^2 + r'^2)^{3/2}}$$

$$E_x = K \cdot x \cdot \int_0^R \frac{\sigma \cdot 2\pi r' dr'}{(x^2 + r'^2)^{3/2}}$$

$$= K \cdot x \cdot \sigma \cdot 2\pi \int_0^R \frac{r' dr'}{(x^2 + r'^2)^{3/2}}$$

$$\text{Let } u = (x^2 + r'^2)^{-\frac{1}{2}}$$

Thus: $du = (-\frac{1}{2})(x^2 + r'^2)^{-3/2} \cdot 2r' dr'$

$$= - \frac{r' dr'}{(x^2 + r'^2)^{3/2}}$$

at $r' = 0$, $u = (x^2)^{-\frac{1}{2}} = \frac{1}{(x^2)^{\frac{1}{2}}} = \frac{1}{|x|}$

at $r' = R$, $u = (x^2 + R^2)^{-\frac{1}{2}}$

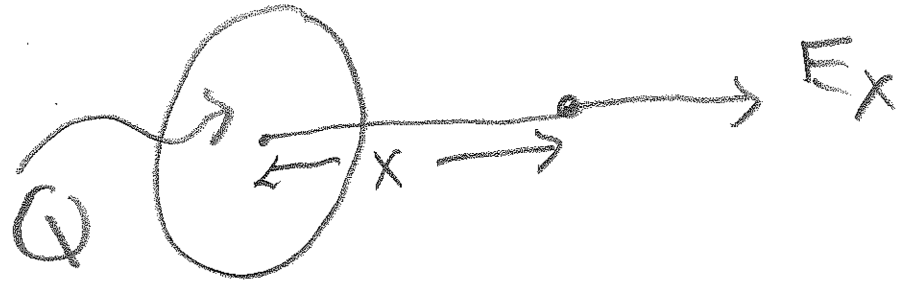
$$= \frac{1}{\sqrt{x^2 + R^2}}$$

$$E_x = K \cdot X \cdot \sigma \cdot 2\pi \int_{1/|x|}^{\frac{1}{\sqrt{x^2+R^2}}} (-du)$$

$$E_x = K \cdot X \cdot \sigma \cdot 2\pi \cdot \int_{\frac{1}{\sqrt{x^2+R^2}}}^{1/|x|} du$$

$$E_x = K \cdot X \cdot \sigma \cdot 2\pi \cdot \left[\frac{1}{|x|} - \frac{1}{\sqrt{x^2+R^2}} \right]$$

[WORKS FOR $x > 0$ OR $x < 0$.]



LET'S ASSUME $x \geq 0$:
DISTRIBUTE x INTO THE BRACKETS.

$$E_x = K \cdot \sigma \cdot 2\pi \cdot \left[\frac{x}{|x|} - \frac{|x|}{\sqrt{x^2 + R^2}} \right]$$

IF $K = \frac{1}{4\pi\varepsilon_0} \Rightarrow K\sigma 2\pi = \frac{1}{4\pi\varepsilon_0} \cdot \sigma \cdot 2\pi$

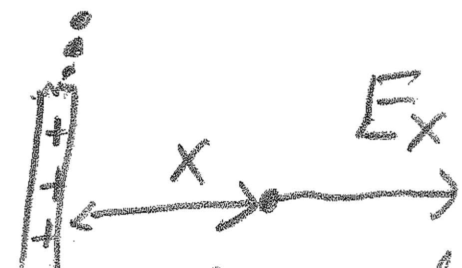
$$E_x = \frac{\sigma}{2\varepsilon_0} \cdot \left[1 - \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}} \right]$$

CHECK 2 LIMITS:

(i) Let $R \to \infty$ and (ii) $x \to \infty$

(i) $$\frac{1}{\sqrt{1 + \frac{R^2}{x^2}}} \Rightarrow 0$$

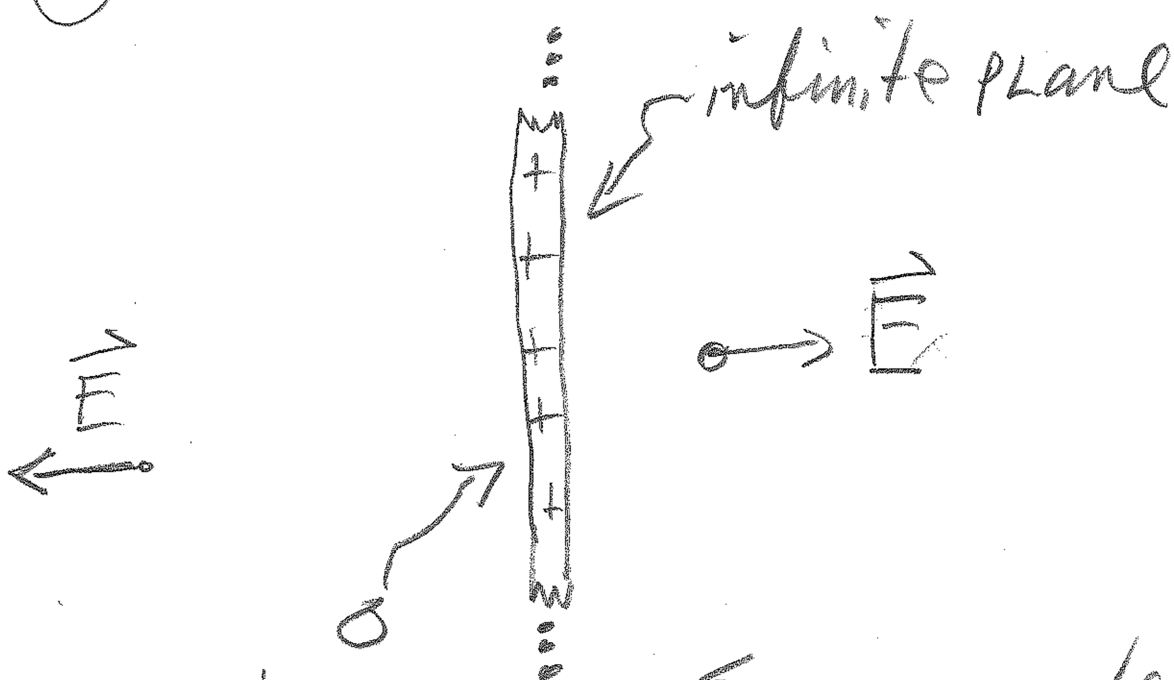
$$E_x = \frac{\sigma}{2\varepsilon_0}$$



E_x due to infinite plane

E_x independent of x.

i



$$|\vec{E}| = E = \frac{\sigma}{2\varepsilon_0} \text{ everywhere!}$$

E = constant!

FIELD lines ARE STRAIGHT.