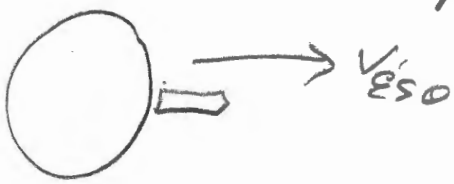
Sample Test 4 \12 Fall

CH13 – Escape speed; potential energy; conservation of energy.

$v_{esc} = ?$

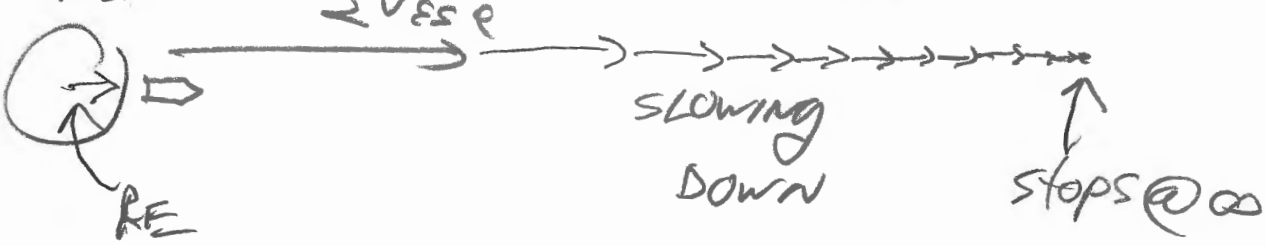
(a)



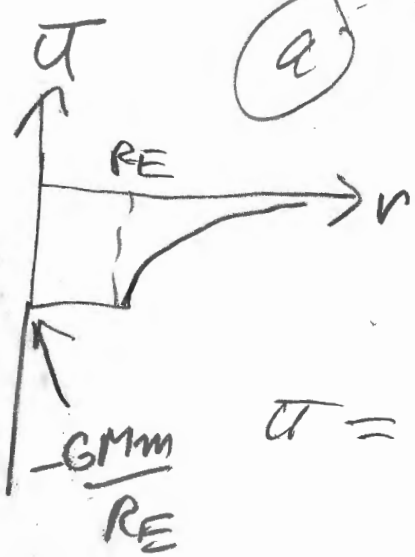
Sample Test 4

(b)

$v_\infty = ?$



(a) Easy warm up problem



$$KE_0 + U_0 = KE_\infty + U_\infty$$

$U_\infty \to 0$

$$U = -\frac{GM_E m}{r}, \quad r \to \infty, \quad U = 0$$

IF $v_0 = v_{esc}$, $KE_\infty = 0$

$$\frac{1}{2}mv_0^2 - \frac{GM_E m}{R_E} = 0 + 0$$

$$\to v_0 = v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = 11.2\,\frac{km}{s} = 11200\,\frac{m}{s}$$

11.2 km/s causes escape for other launch directions

(b)

$$v_0 = 2v_{esc}^{*}$$

$$v_{esc}^{*} = \sqrt{\frac{2GM_E}{R_E}}$$

$$2 \cdot v_{esc} = 2 \cdot \sqrt{\frac{2GM_E}{R_E}}$$

$$= \sqrt{\frac{4 \cdot 2GM_E}{R_E}}$$

$$= \sqrt{\frac{8GM_E}{R_E}}$$

$$\boxed{v_0 = \sqrt{\frac{8GM_E}{R_E}}}$$

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R_E} = KE_\infty + U_\infty$$

U_0

$$\boxed{v_0 = 2 \cdot \sqrt{\frac{2GM_E}{R_E}}}$$

$$U_\infty = -\frac{GM_E m}{r}$$

$$\frac{1}{2}m \cdot 4v_{esc}^2 - \frac{GM_E m}{R_E} = KE_\infty$$

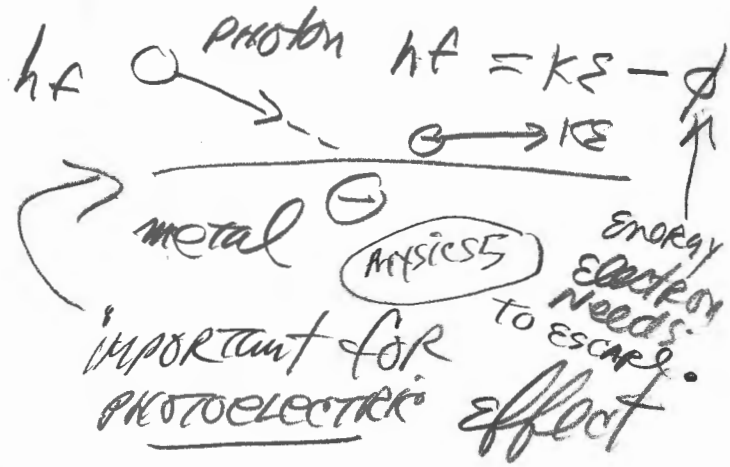
$r = \infty$

$$\frac{1}{2}m\,4 \cdot \frac{2GM_E}{R_E} - \frac{GM_E m}{R_E} = KE_\infty \qquad = 0$$

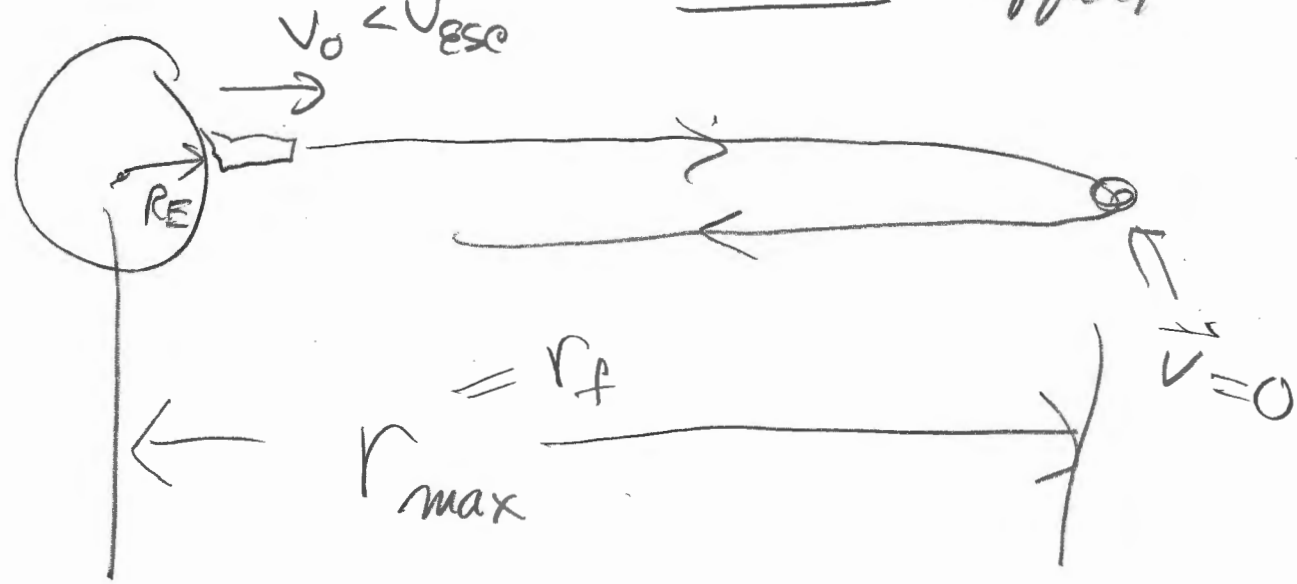
$$\frac{4GM_E m}{R_E} - \frac{GM_E m}{R_E} = KE_\infty$$

$$\frac{3GM_E m}{R_E} = \frac{1}{2}mv_\infty^2 \Rightarrow \sqrt{\frac{6GM_E}{R_E}} = v_0$$

$v_\infty < v_0$



What if?



$r_{max} = ?$

Make observations

(i)

$$KE_0 + U_0 = KE_f + U_f$$

$$\frac{1}{2}mv_0^2 - \frac{GM_E m}{R_E} = 0 - \frac{GM_E m}{r_f}$$

solve for $r_f = r_{max}$.

$$\frac{1}{2}mv_0^2 - \frac{GM_E m}{R_E} = 0 - \frac{GM_E m}{r_f}$$

set $v_0 = \frac{v_{esc}}{2}$

$$\frac{1}{2}m\frac{1}{4}\cdot v_{esc}^2 - \frac{GM_E m}{R_E} = 0 - \frac{GM_E m}{r_f}$$

$$\frac{1}{2}m\cdot\frac{1}{4}\cdot\frac{2GM_E}{R_E} - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_f}$$

$$\frac{GM_E m}{4R_E} - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_f}$$

$$-\frac{3}{4}\frac{\cancel{GM_E m}}{R_E} = -\frac{\cancel{GM_E m}}{r_f}$$

$$\frac{3}{4R_E} = \frac{1}{r_f}$$

$$r_f = \frac{4}{3}R_E > R_E = 6.4\times10^3\,km$$

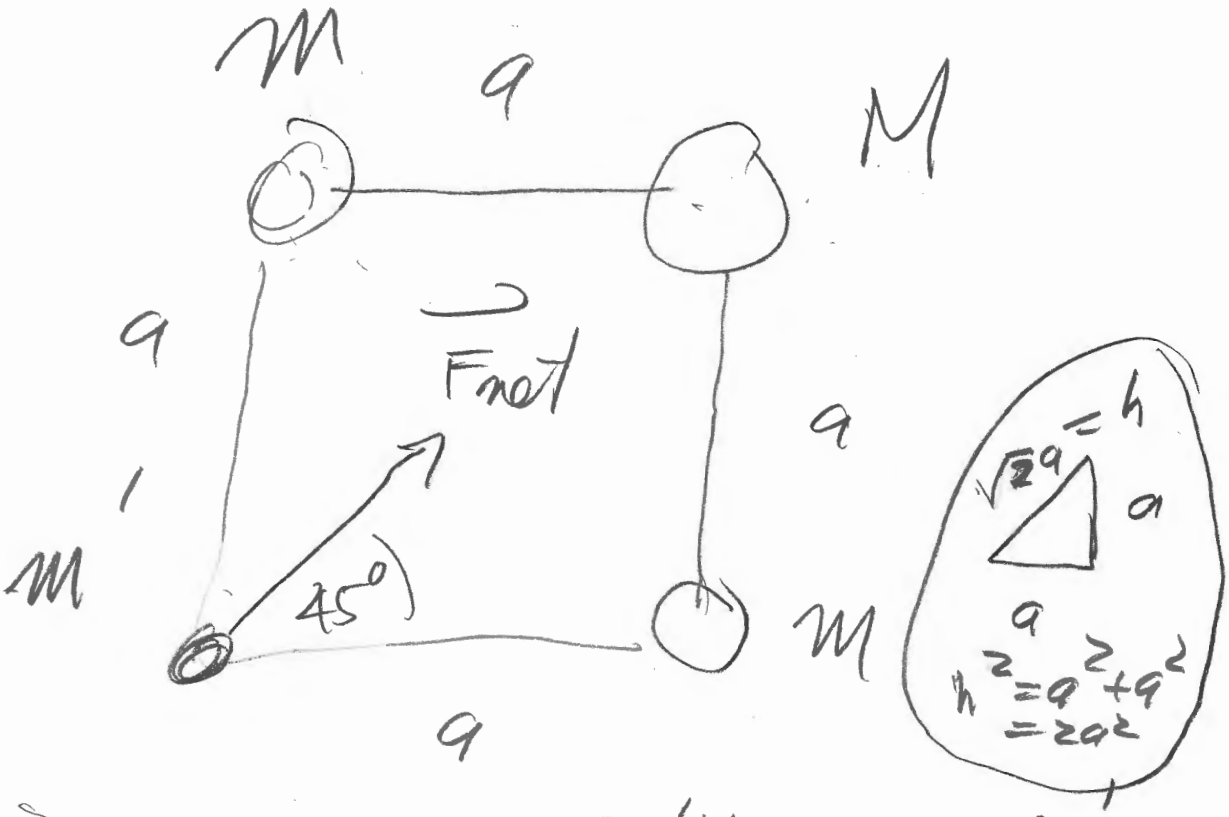
physically expected

CH 13 LINKS:

"sample final" on main

syllabus page (A)

Ch 13 (43)



$$F_{x_{net}} = 0 + \frac{Gm'M}{(\sqrt{2}a)^2} \cdot \cos 45 + \frac{Gmm}{a^2}$$

$$F_{y_{net}} = \frac{Gm'm}{a^2} + \frac{Gm'M}{(\sqrt{2}a)^2} \cdot \sin 45 + 0$$

Note: $F_{x_{net}} = F_{y_{net}} \Rightarrow F_{net} = \sqrt{2} \cdot F_{y_{net}}$

$= \sqrt{2}\, F_{x_{net}}$

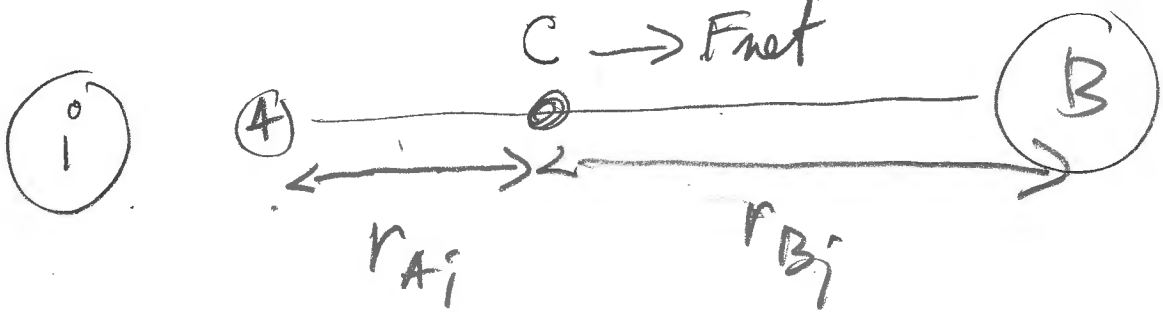
ch13

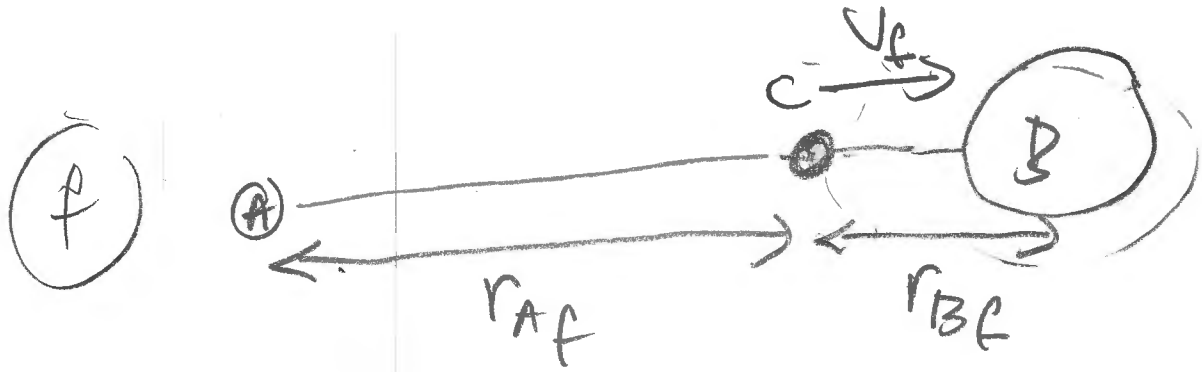
62

Notes:

$F_{right} = G\cdot(0.1)\cdot\left[\frac{10}{0.10^2} - \frac{5}{0.4^2}\right] > 0$

$C \rightarrow F_{net}$





$$KE_i + U_i = KE_f + U_f$$

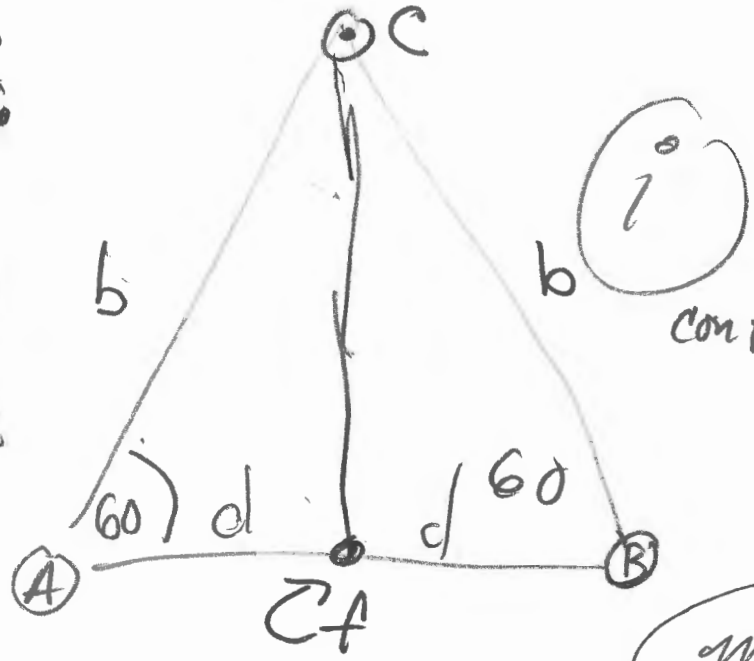
$$0 - \boxed{\left(\frac{Gm_c m_A}{r_{A_i}} + \frac{Gm_c m_B}{r_{B_i}}\right)} = \frac{1}{2} m_c v_f^2 - \left(\frac{Gm_c m_A}{r_{A_f}} + \frac{Gm_c m_B}{r_{B_f}}\right)$$

find v_f

given $r_{A_i}, r_{B_i}, r_{A_f}, r_{B_f}$

WHAT IF?

Isosceles Triangle

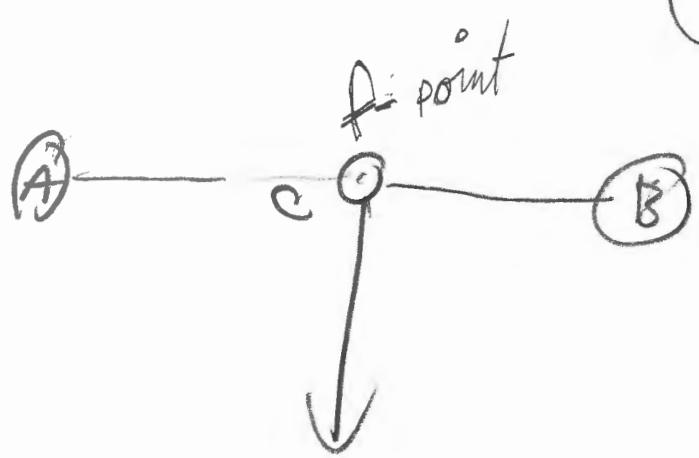


i configuration

$m_A = m_B$

triple STAR system

f configuration



$v_f = ?$

$$KE_i + U_i = KE_f + U_f$$

$$0 + \left(-\frac{2Gm_c m_A}{b}\right) = \frac{1}{2} m_c v_f^2 + \left(-\frac{2Gm_c m_A}{d}\right)$$

CH 9 THEMES

ALREADY DISCUSSED (TEST 3)

$v_t = r \cdot \omega$

and other relations

$\boxed{a_c = \omega^2 r} = \frac{v_t^2}{r}$ between linear, rotational

$a_t = r \cdot \alpha$ variables

1. motion
2. Energy
3. moment of inertia I

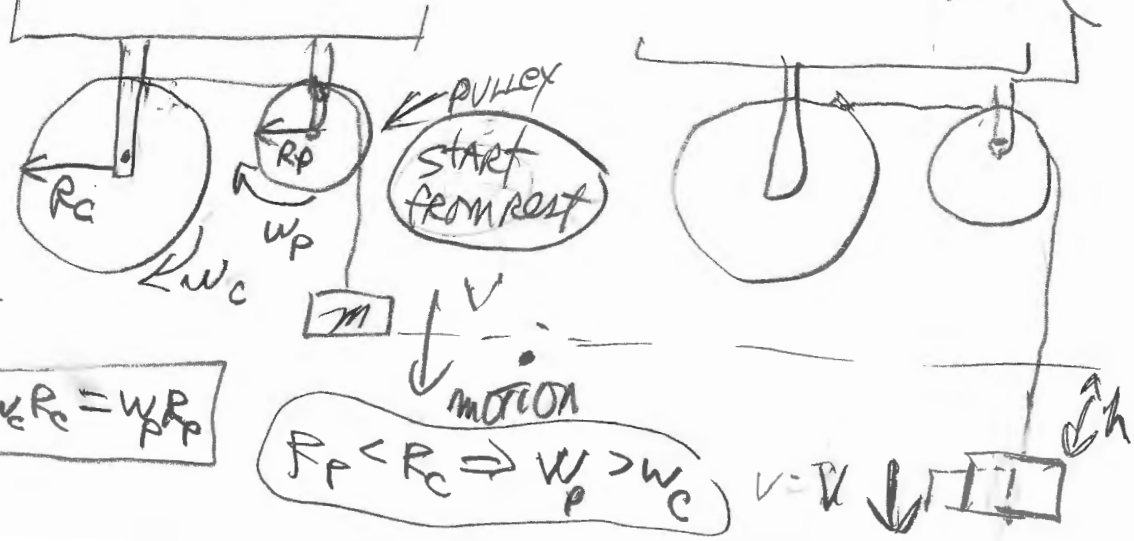
CH 9 SUMMARY problem

sample TEST 4 LINK at CH. 9 QUIZ PAGE

#4

pulley spins faster

START: REST



$\boxed{v = \omega_c R_c = \omega_p R_p}$

$R_p < R_c \Rightarrow \omega_p > \omega_c$

CH9

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_p\omega_p^2 + \frac{1}{2}I_c\omega_c^2$$

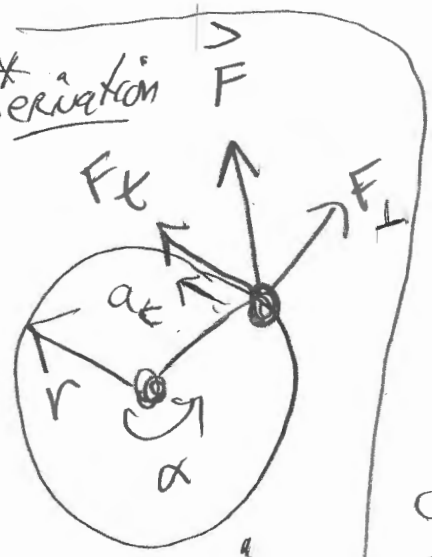
$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{1}{2}M_pR_p^2\frac{v^2}{R_p^2} + \frac{1}{2}\cdot\frac{1}{2}M_cR_c^2\frac{v^2}{R_c^2}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}M_pv^2 + \frac{1}{4}M_cv^2$$

FIND v given h OR VICE VERSA.

CH10 Torque

*Derivation



a_t = tangential acceleration

$ma_t = F_t$

$m\alpha \cdot r = F_t$

$m\alpha r^2 = r \cdot F_t$

$mr^2\alpha = \tau$; $I = mr^2$

$|\vec{\tau}| = r_\perp \cdot F$

$|I\alpha| = r_\perp F$ *

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$|\vec{L}| = r_\perp \cdot p$

$p = |m\vec{v}|$

Solid objects: $L_z = I\omega$

TABLE 9.2

Final May 29th

CH8, 9, 10, 11, 13, 14

CH8	uses	concepts	from	CH7, CH6
CH9	"	"	"	CH2, CH7
→ CH10	"	"	"	CH2, CH9, CH4, CH5
→ CH11	"	"	"	CH10, CH4, CH5
→ CH13	"	"	"	CH7, CH3, CH5
→ CH14	"	"	"	CH7; CH4,5

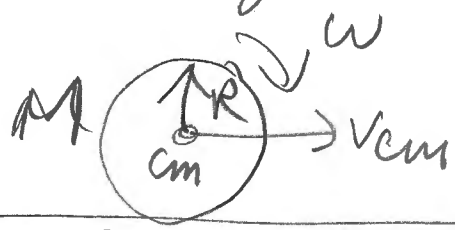
CH10 THEMES: $|\vec{\tau}| = r_\perp F$;

$|\vec{\tau}| = |I \cdot \alpha|$; rolling w/o slipping; conservation of Energy;

ETC → more themes in Lab2 this week

CH10

Rolling w/o slipping



$$\boxed{v_{cm} = \omega R}$$

Defines rolling w/o slipping

$f_s = 0$

horizontal

$$KE = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\frac{v_{cm}^2}{R^2}$$

Typically: $I = \underbrace{\frac{1}{2}MR^2}_{\text{Disk (wheel)}}$ or $\underbrace{\frac{2}{5}MR^2}_{\text{Ball (solid)}}$

Rolling w/o slipping

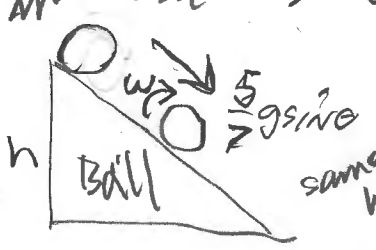
$v_{cm} = \omega R$

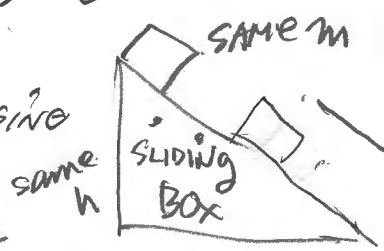
Example 10.7

Because of rolling

$$a_{cm} = \frac{5}{7} g \sin\theta < g\sin\theta$$

Face





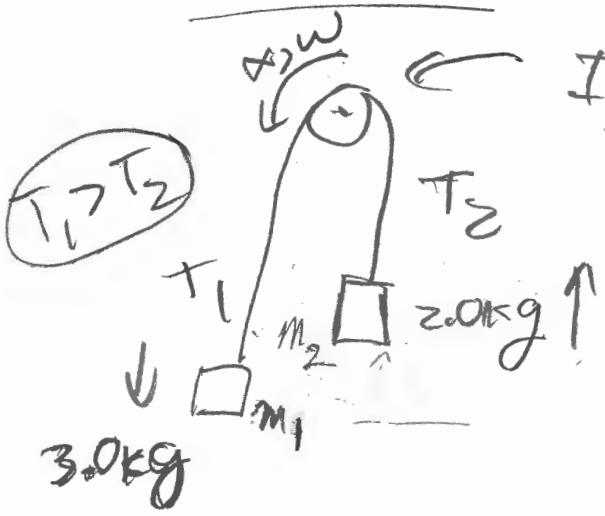
CH10

(CH 4, 5)

other MAJOR CHO THEME:

conservation of $\vec{L}$.

"final sp05" LINK



Given:

$a = 0.02\ m/s^2$

$T_1 = ?$

$T_2 = ?$

$I = ?$

$R = 1.0\ (m)$

Equations

a = magnitude of common acceleration

(A) $m_1 a = m_1 g - T_1$

(B) $m_2 a = T_2 - m_2 g$

$I = ?$
$T_1 = ?$
$T_2 = ?$

GENERALLY

3 UNKNOWN:
T_1, T_2, a
TYPICAL

$\sum \tau = \circlearrowleft - \circlearrowright$

$I \cdot \alpha = R \cdot T_1 - R T_2$

$I \frac{a}{R} = R(T_1 - T_2)$ $\left(\alpha = \frac{a}{R}\right)$ $\Rightarrow$ $\boxed{I \frac{a}{R^2} = T_1 - T_2}$ (C)

LINE of action

$\tau_1 = r_\perp T_1$

$= R T_1$

T_1

typically if a, T_1, T_2 are unknown:

ADD EQuation to get a:

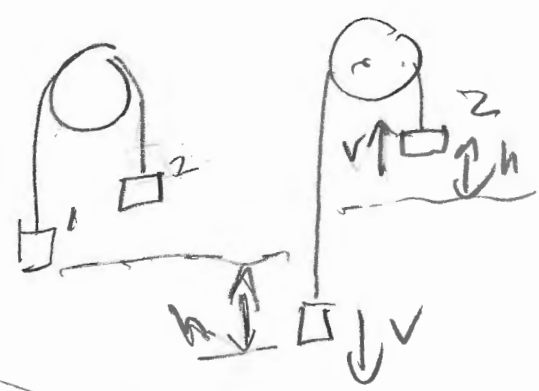
$$a\cdot\left(\frac{I}{R^2}+m_1+m_2\right) = m_1 g - m_2 g$$

$$a = \frac{(m_1-m_2)g}{\left(\frac{I}{R^2}+m_1+m_2\right)}$$

$$\left.\begin{aligned} m_1 a &= m_1 g - T_1 \\ m_2 a &= T_2 - m_2 g \end{aligned}\right\} \text{FIND } T_1, T_2$$

now get: $v^2 = 2ah$ (CH2)

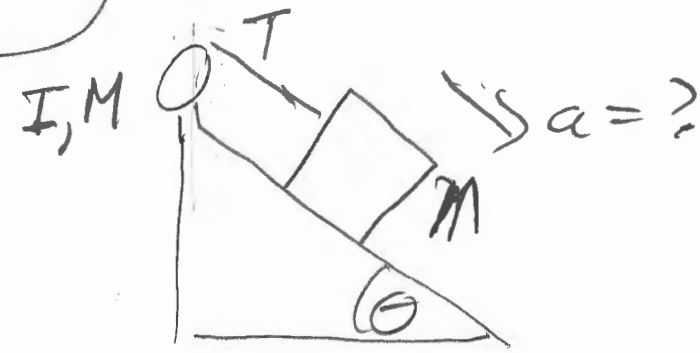
VERTICAL RISE



FIND v IF STARTS FROM REST

CWCO

#70



Review this + ADD a "WHAT IF"

JUST like #1 Test 4

(1.) $ma = mg\sin\theta - T$

$ma = mg\sin\theta - T$

(2.) $I\frac{a}{R} = R \cdot T \Rightarrow I\frac{a}{R^2} = T$

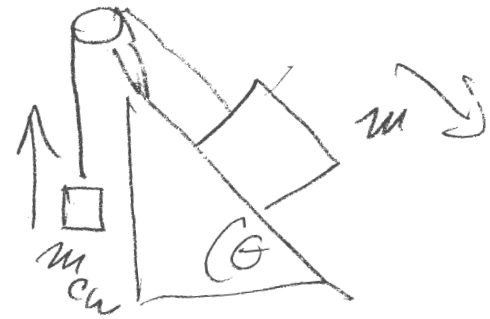
$a\left(\frac{I}{R^2} + m\right) = mg\sin\theta$

$a = \dfrac{mg\sin\theta}{\left(\frac{I}{R^2} + m\right)}$

What if:

WHAT IF

3 EQUATIONS



(1.) $ma = mg\sin\theta - T_1$

(2.) $I\frac{a}{R^2} = T_1 - T_2$

(3.) $m_{cw}a = T_2 - m_{cw}g$

ADD: cancel OUT T_1, T_2.

ADD: $a\left(m + \frac{I}{R^2} + m_{cw}\right) = mg\sin\theta - m_{cw}g$

$$a = \frac{mg\sin\theta - m_{cw}g}{\left(m + \frac{I}{R^2} + m_{cw}\right)}$$

GO BACK FIND T_1, T_2

(EASY)

CH 11 see CH 11 SUMMARY page 3.59.

THEMES:

1. beam, cable problems simulating "arm and muscle" (Bio-physics Bio-engineering)

2. Ladder problems (see expanded discussion of example 11.3)

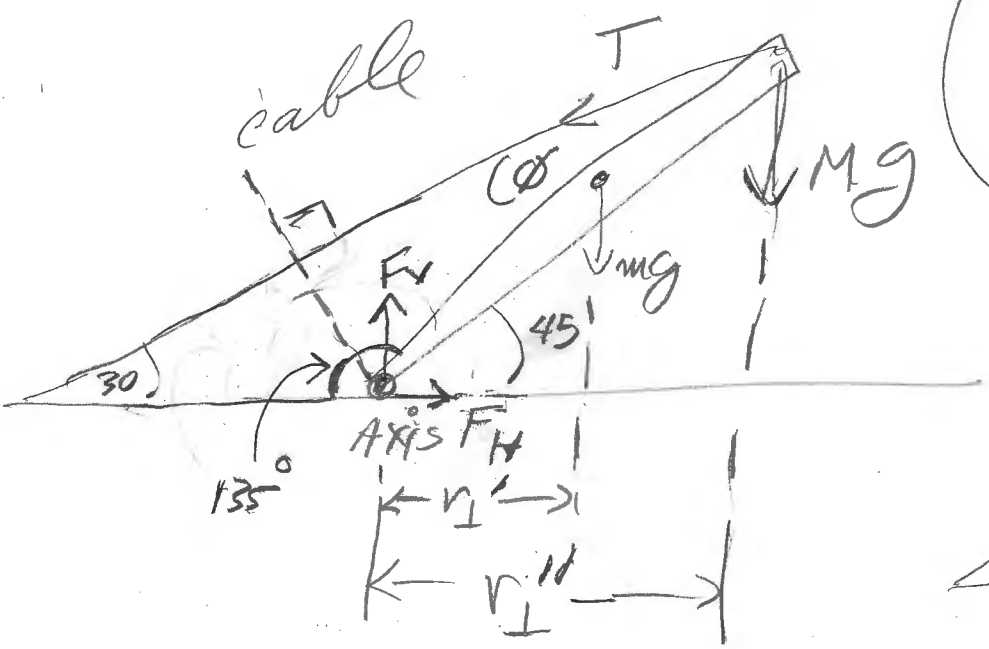
3. Basic questions about (multi-choice section) stress, strain.

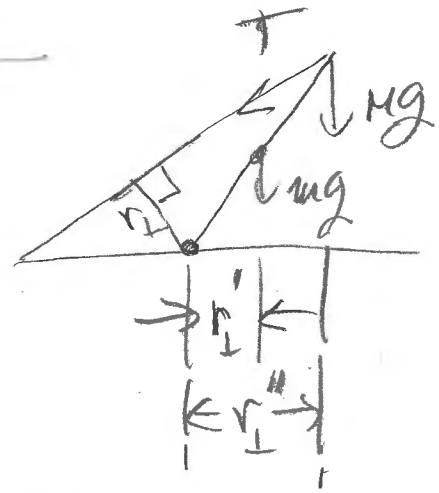
CH11

(13) (b) SIMPLIFIED VERSION

$\phi + 30 + 135 = 180$

$\phi = 15^\circ$





$\sum \tau = \circlearrowleft - \circlearrowright$

$0 = r_\perp T - r_\perp' mg - r_\perp'' Mg$

$= (l \sin 15) \cdot T - \left(\frac{l}{2} \cdot \cos 45\right) \cdot mg - (l \cos 45) \cdot Mg$

$\Longrightarrow$ solve T.

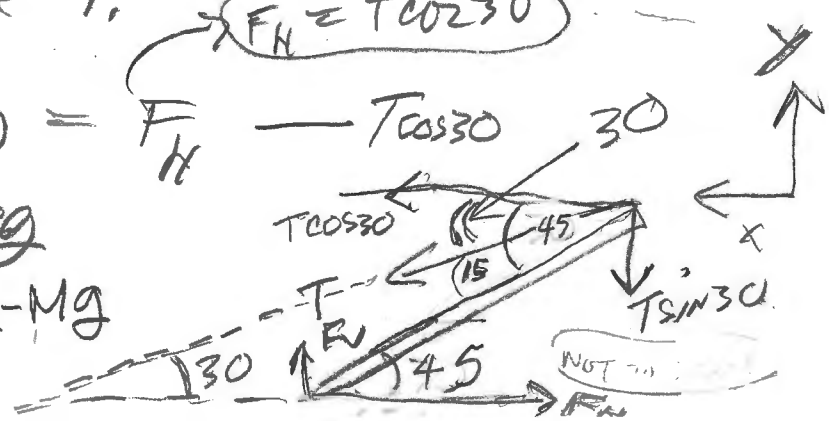
$F_H = T\cos 30$

$\sum F_x = 0 = F_H - T\cos 30$

$\sum F_y = \text{pos} - \text{neg}$

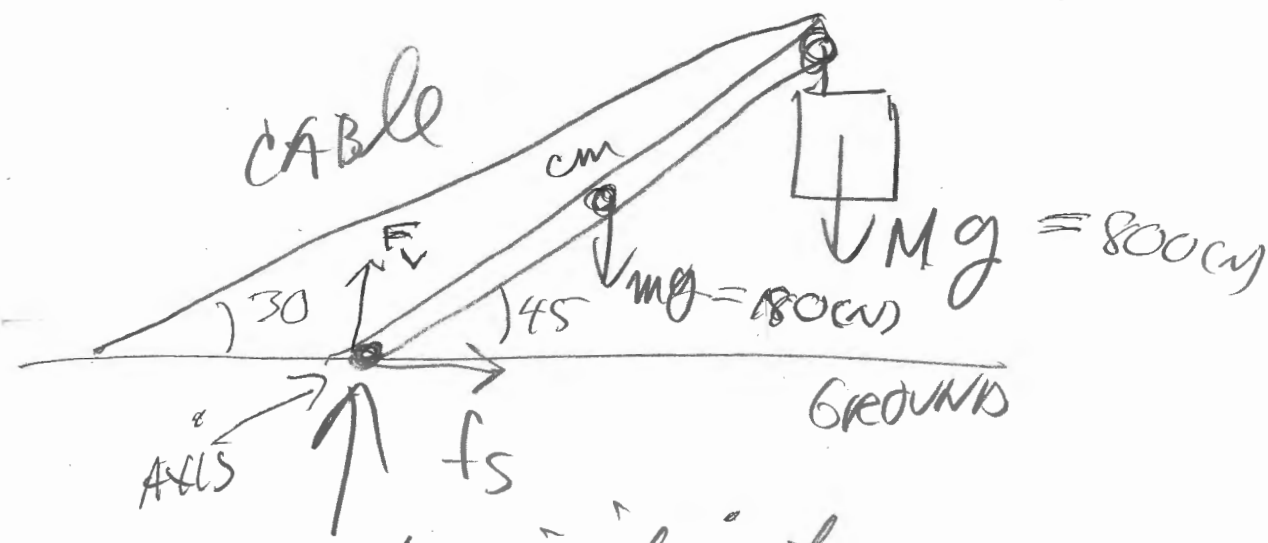
$0 = F_V - T\sin 30 - mg - Mg$

$F_V = T\sin 30 + (m + M)g$



Ch 11 Follow up to 13 (b)

pretend you have a ladder: LET LOAD $Mg = 800(N)$
Let Beam $mg = 180(N)$



not a rigid pivot.

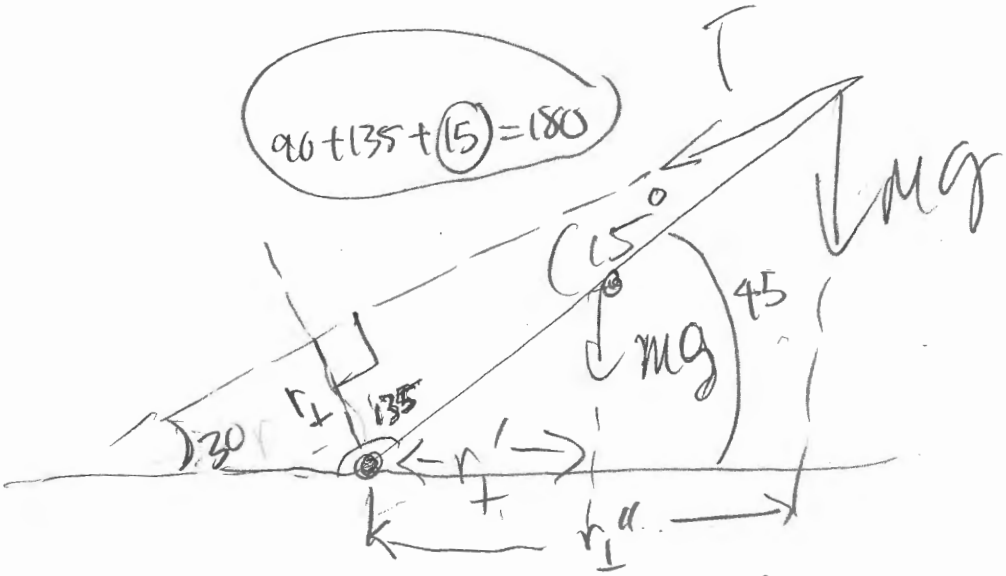
Given m and M, will the ladder slip?

ASSUME $\mu_s = 0.32$.

CHECK: $f_s \geq f_{s_{max}} = \mu_s \cdot F_V$

F_V = NORMAL FORCE at GROUND (AXIS)

13(b) follow up — LADDER SCENARIO.



$$0 = r_\perp \cdot T = r_\perp' \cdot mg - r_\perp'' \cdot Mg$$

$$T = \frac{r_\perp' mg + r_\perp'' Mg}{r_\perp}$$

$$= \frac{\frac{\ell}{2} \cdot \cos 45 \cdot (180) + \ell \cdot \cos 45 \, (800)}{\ell \cdot \sin 15}$$

$$= \frac{(180)\frac{\sqrt{2}}{4} + 800 \cdot \frac{\sqrt{2}}{2}}{\sin 15}$$

$$= \frac{\frac{(180)(0.707)}{2} + (800)(0.707)}{0.26}$$

$$= 2396 (N)$$

will it slip? yes!
see below:

$f_s = T \cos 30$

$f_s = (2396)(0.867)$

$f_s = \boxed{2077 (N)}$

$f_{s_{max}} = (0.32) F_N = (0.32)(2180) = \boxed{698 (N)}$

$F_N = T \sin 30 + 980$

$F_N = 1200 + 980 = \boxed{2180}$

CH 14

Physical pendulum and apply to CH10, CH8

CH14

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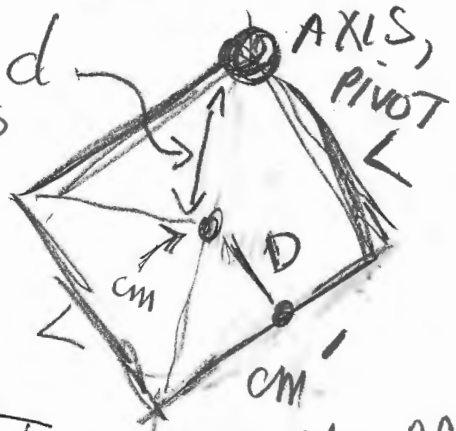


4 RODS

m = TOTAL MASS

$\omega = \sqrt{\frac{mgd}{I_{AXIS}}}$

$d = \frac{\sqrt{2} \cdot L}{2}$

$d = \frac{L}{\sqrt{2}}$

$I_{cm} = 4\left(\frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2\right)$

$= I_{cm} + Md^2$

PIVOT = axis at REST

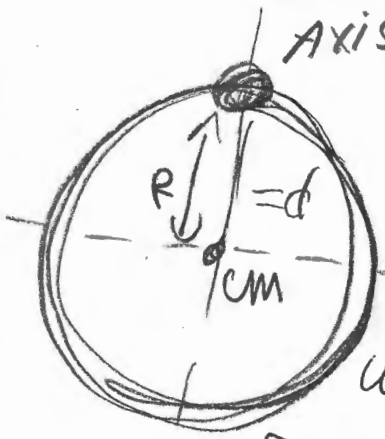
M = ROD MASS

m = TOTAL MASS

$I_{AXIS} = I_{cm} + Md^2$

$\boxed{m = 4 \cdot M}$

52.



$\omega = \sqrt{\frac{mgd}{I_{AXIS}}}$

$\omega = \sqrt{\frac{mgR}{mR^2 + MR^2}}$

$\omega = \sqrt{\frac{g}{2R}}$

$\boxed{I_{AXIS} = I_{cm} + md^2}$

$\omega = \sqrt{\frac{mgd}{I_{AXIS}}}$

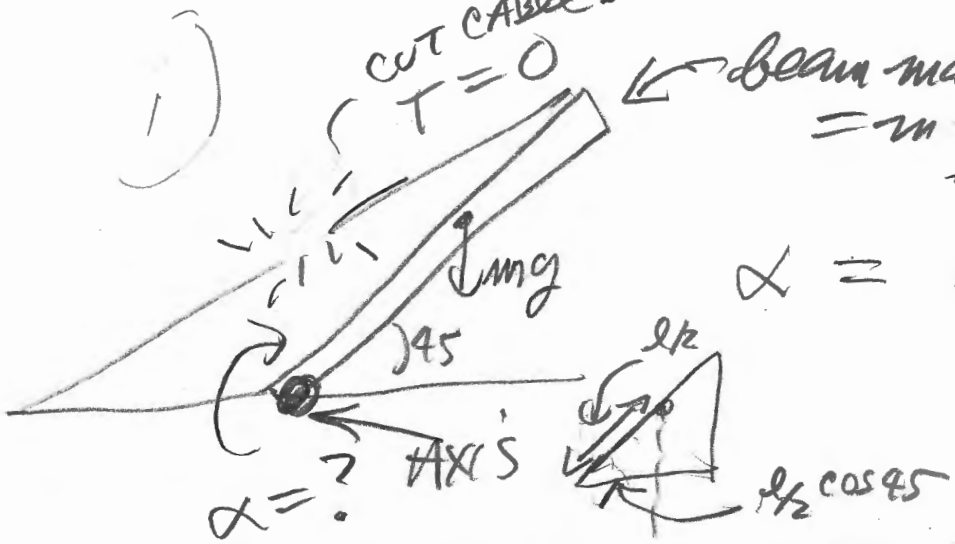
$T = 2\pi \sqrt{\frac{I_{AXIS}}{mgd}}$

Additional CH 9, 10, 11 related issues

(i) #13(b) FOLLOW UP

cut cable and remove load mass at end (remove M)

(i)

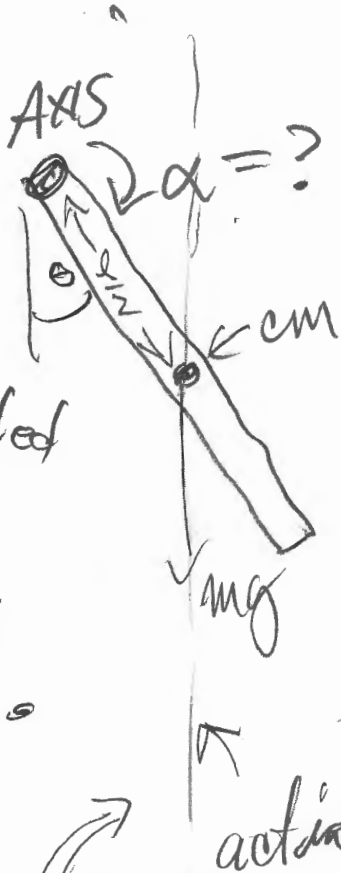


beam mass $= m$; length $= l$.

$$\alpha = \frac{\tau}{I} = \frac{\left(\frac{l}{2}\cos 45\right) mg}{ml^2/3}$$

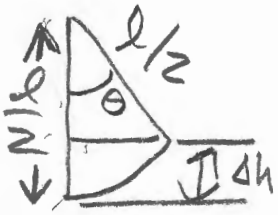
$$\alpha = \frac{3}{2}\frac{g}{l}\cos 45$$

(ii)



$\theta = 45$

$$\alpha = \frac{\tau}{I}$$



$\Delta h = \frac{l}{2}(1-\cos\theta)$

$$= r_\perp mg$$

$$= \frac{\left(\frac{l}{2}\cdot\sin\theta\right)mg}{\frac{ml^2}{3}}$$

set $\theta = 45$

$$= \frac{3}{2}\frac{g}{l}\sin 45$$

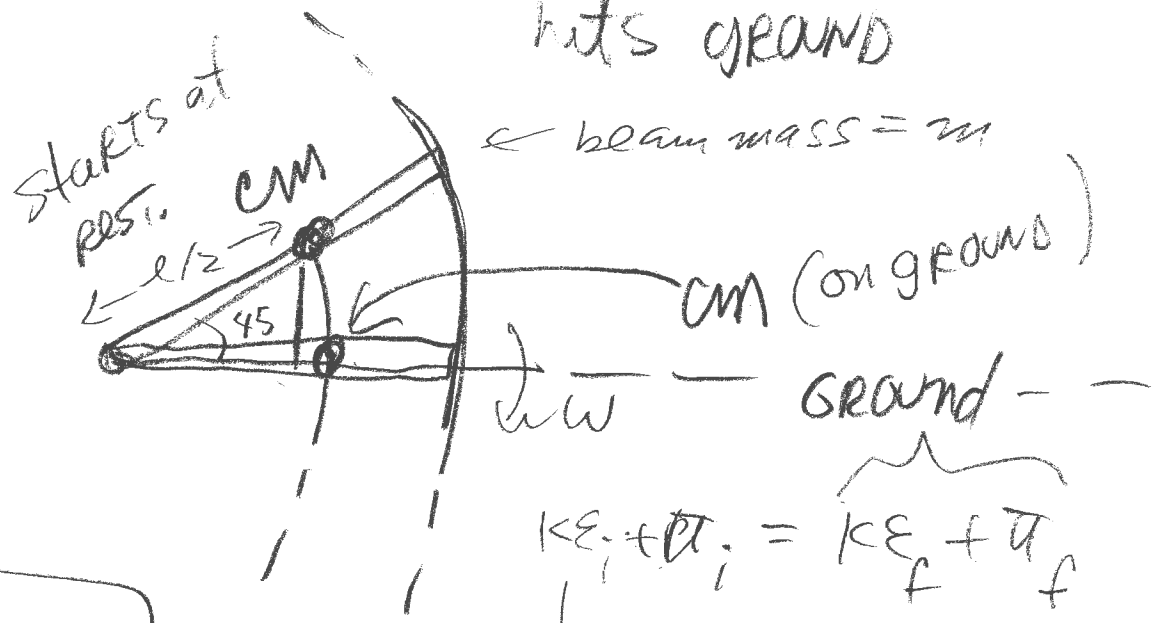
uniform rod suspended at top end, θ = angle w/ vertical. Length $= L$, mass $= M$.

Energy side bar: $\Delta KE + \Delta U = 0$; $I = ml^2/3$

Find ω at bottom. $\frac{1}{2}I\omega^2 - mg\frac{l}{2}(1-\cos\theta) = 0$; $\theta = 45°$

13 b

<u>follow-up</u>: FIND ω when beam hits GROUND



$$KE_i + U_i = KE_f + U_f$$

$$0 + mg\frac{\ell}{2}\sin 45 = \frac{1}{2}I\omega^2 + 0$$

$$mg\frac{\ell}{2}\sin 45 = \frac{1}{2}\cdot\left(\frac{mR^2}{3}\right)\omega^2$$

$$\rightarrow \omega^2 = \frac{3g\sin 45}{\ell}$$

$$\boxed{U = mgY_{cm}}$$