

5-10-13
5-8-13

Test 9 next week take home

on CH 10, 11, 14, and possibly B.
↑ ? ↓

Final May 20th (Wed)

arrange a 3-hour

time period:

1:30 - 4:30

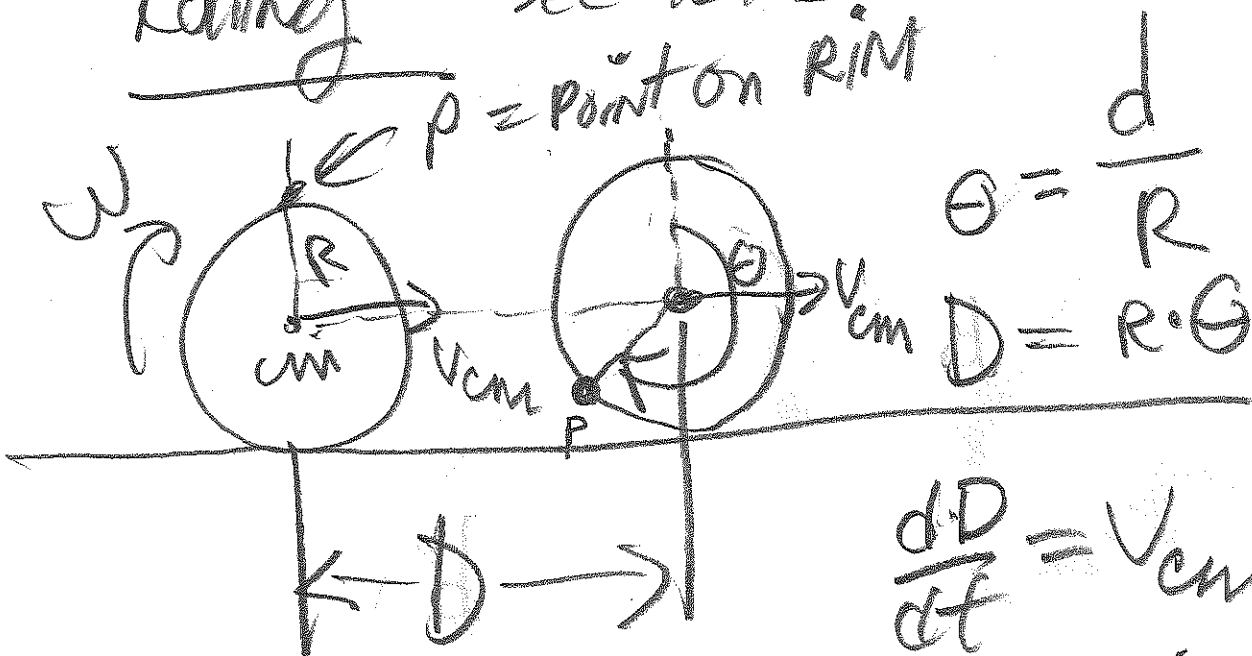
2 - 5 PM

RMTBA (1803?)

Start CH10, CH11, Review

CH10 BASIC cheat sheet
(continued)

Rolling Sec 10.3
 $P = \text{point on RIM}$



$$\theta = \frac{d}{R}$$
$$D = R \cdot \theta$$

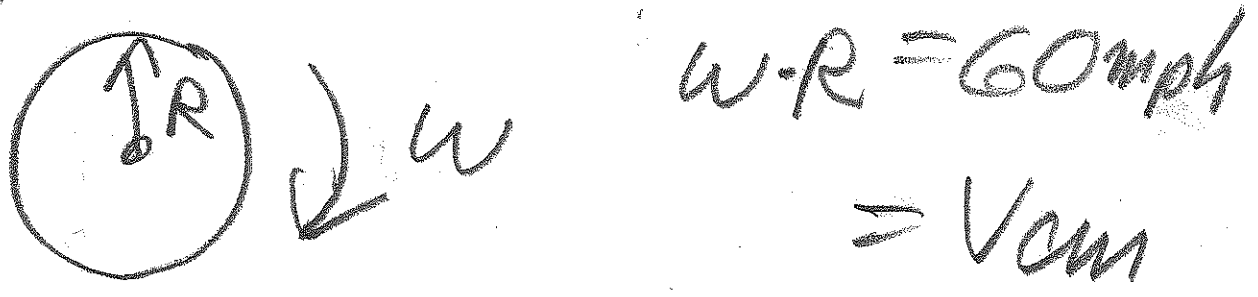
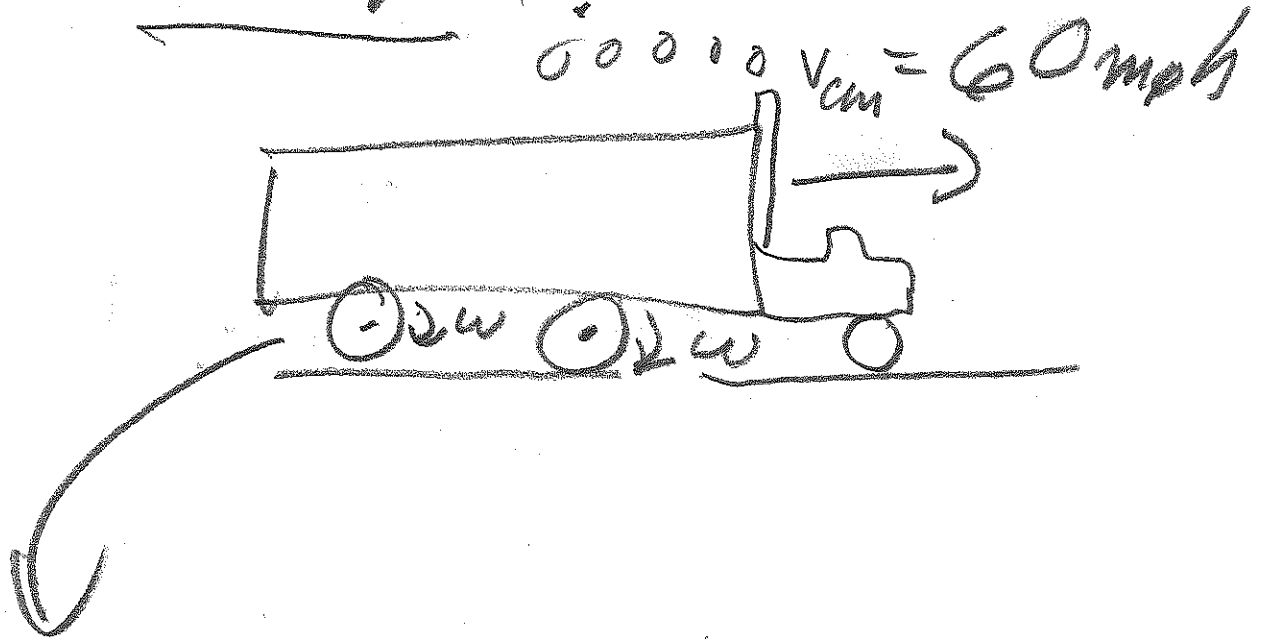
$$\frac{dD}{dt} = v_{cm}$$

= linear speed of C.M.

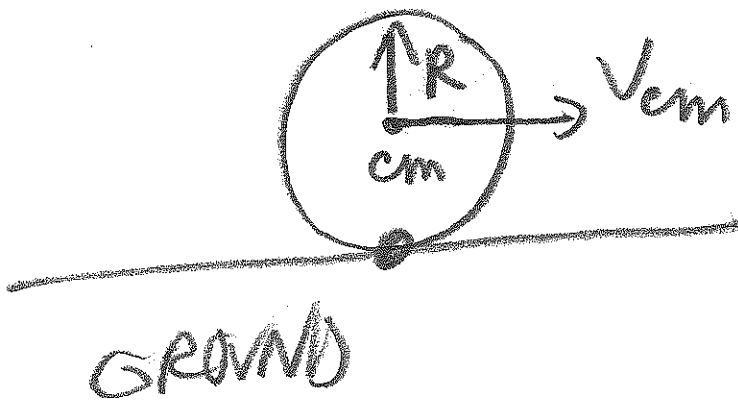
$$\frac{dD}{dt} = R \frac{d\theta}{dt}$$

$v_{cm} = R \cdot \omega$ Rolling w/o slipping

Example: Truck on S80



ENERGY of ROLLING : 2 Reference frames:



- (A) at rest relative TO GROUND
- (B) at rest relative to the cm.

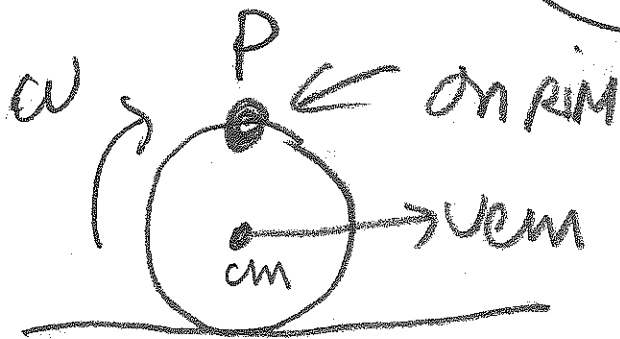
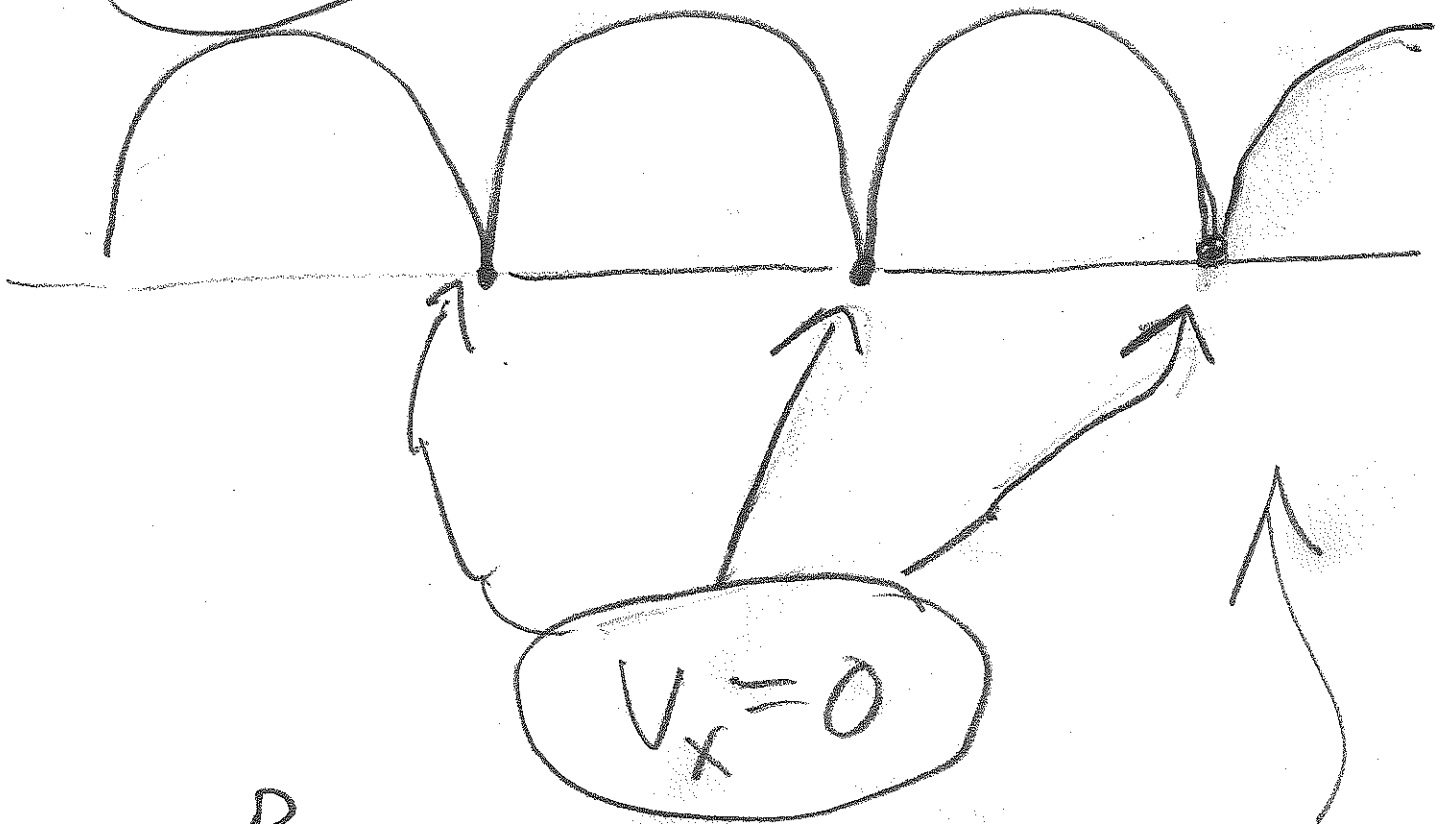
OBSERVER ON GROUND:

PLOT PATH OF A

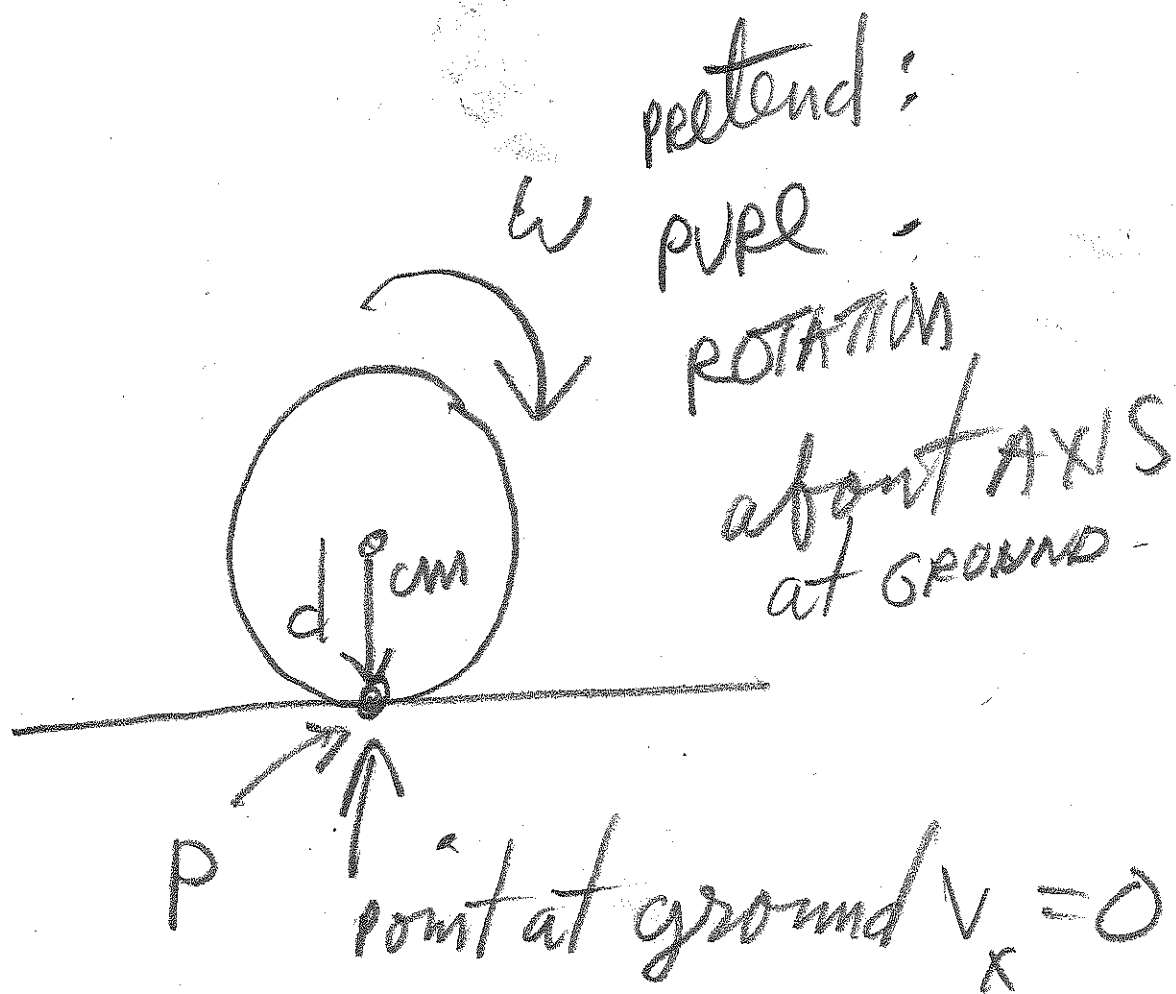
POINT ON THE RIM:

icycloid

$\rightarrow x$ (POS)



PLOT OF P
ON RIM AS
WHEEL TURNS



$$KE = \frac{1}{2} I_P \omega^2$$

Parallel-Axis Thm: $I_P = I_{cm} + Md^2$

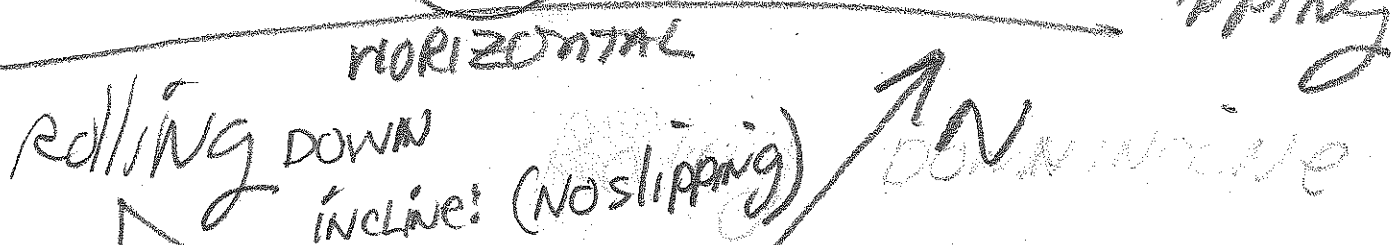
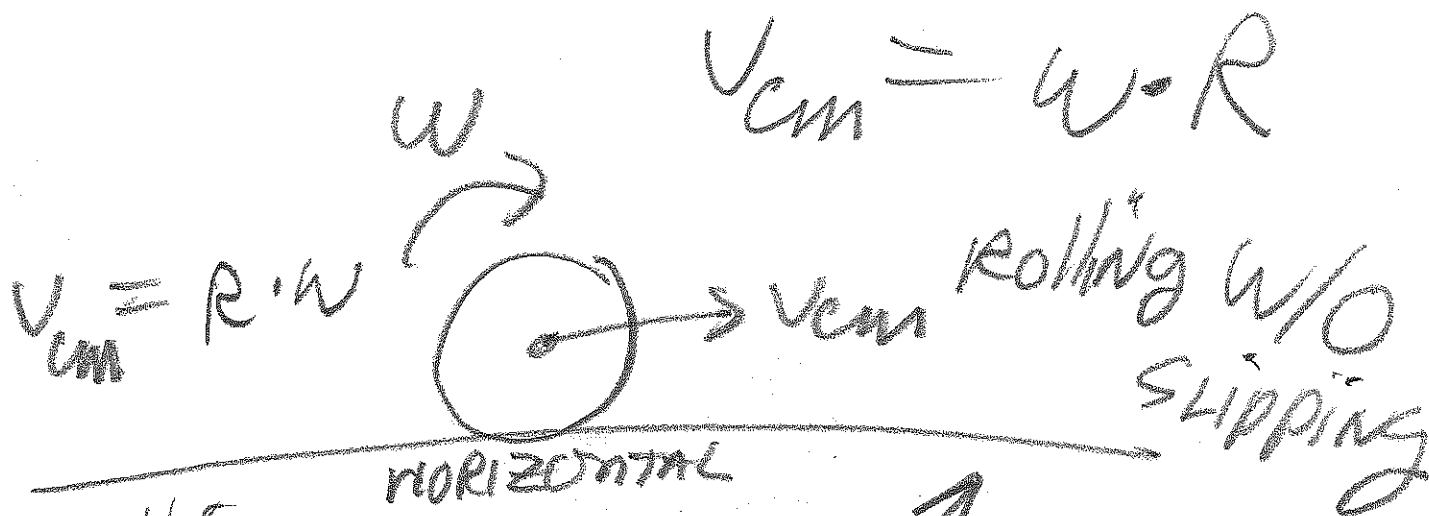
but $d = R = \text{radius}$

$$KE = \frac{1}{2} (I_{cm} + MR^2) \omega^2$$

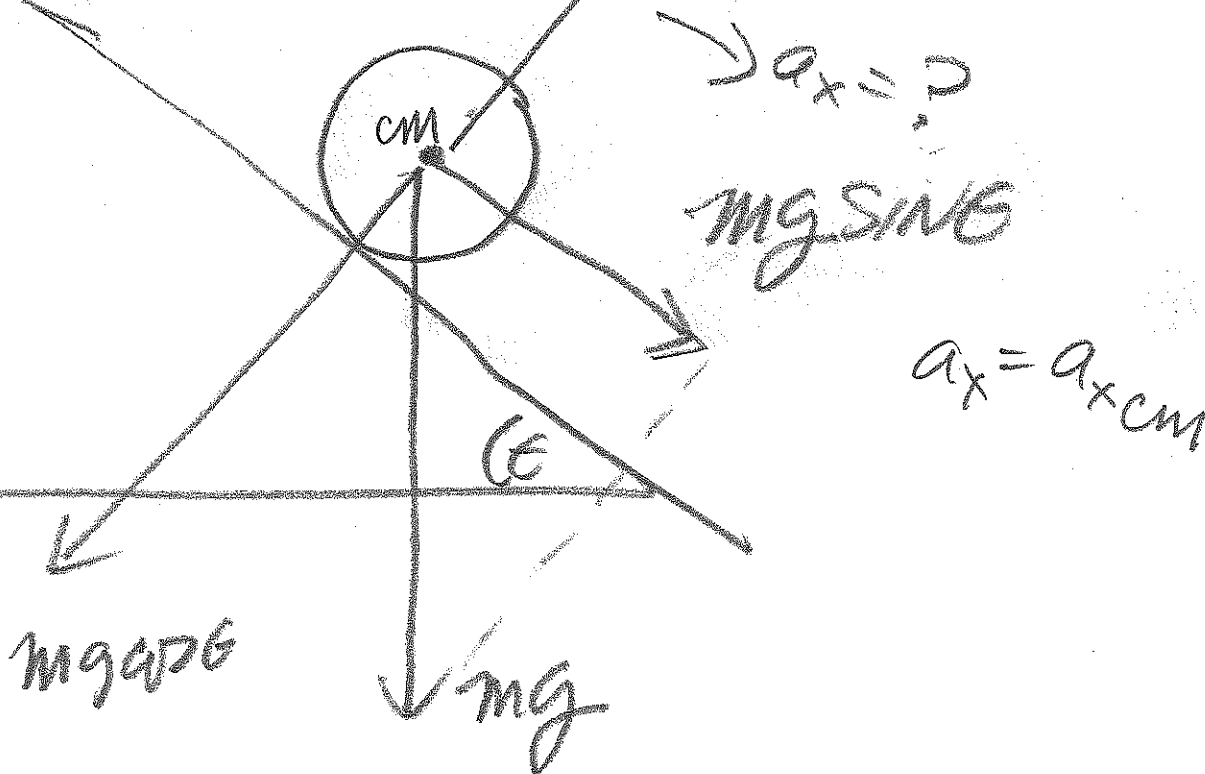
$$= \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} MR^2 \omega^2$$

$$KE = KE_{\text{ROT}} + KE_{\text{TRAN}}$$

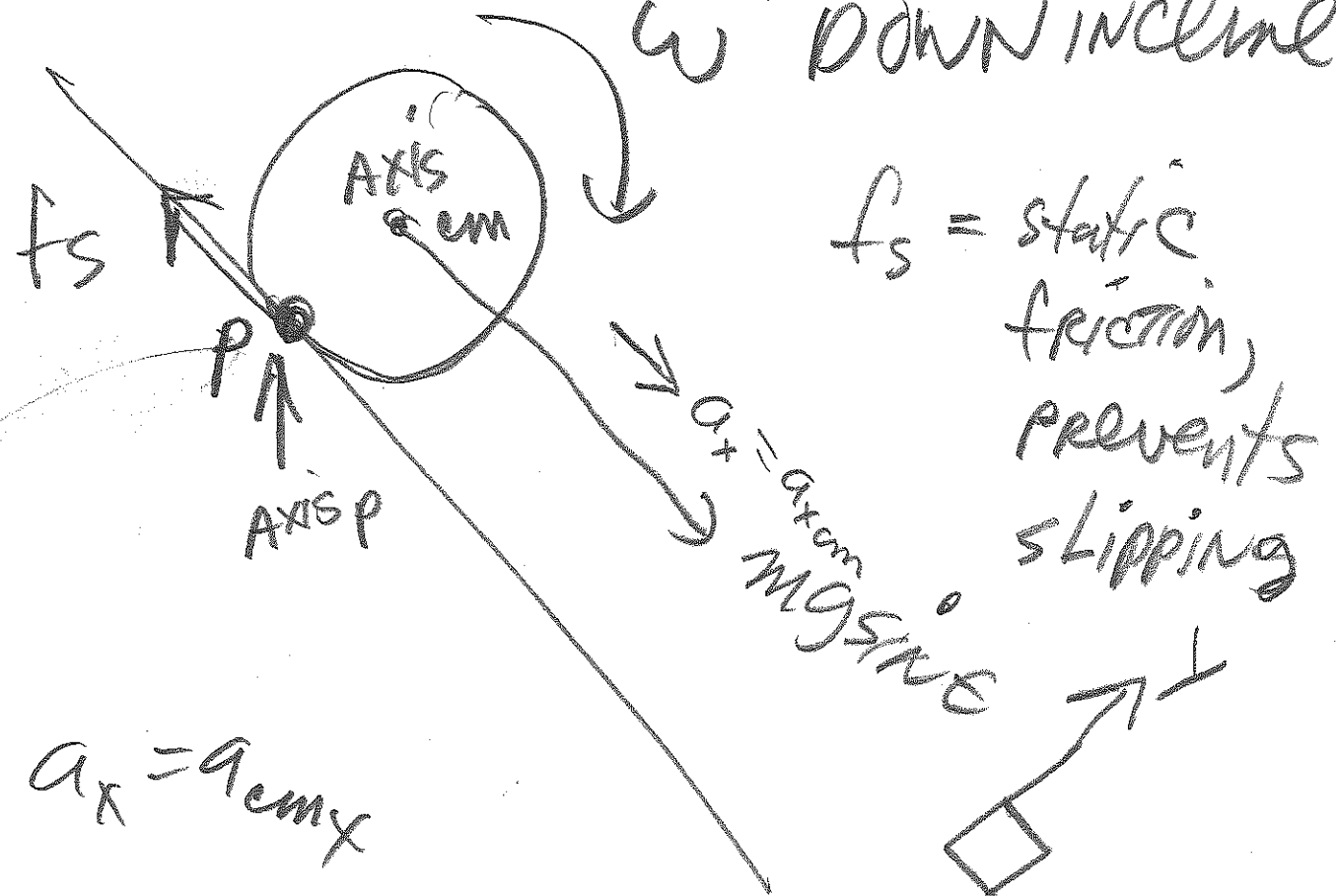
$$= \frac{1}{2} I_{\text{CM}} \omega^2 + \frac{1}{2} M V_{\text{CM}}^2$$



- Example 10-7
- fig 10.20



Roll w/o slipping
 w DOWN INCLINE



$a_x = a_{cmx}$

$\Sigma F_x = \text{pos} - \text{neg}$

Translation: $ma_x = mg \sin \theta - f_s$

UNKNOWN

a_x, f_s



$\tau = I \cdot \alpha$ and $\alpha = a_x / R$

$r \cdot f_s = I \cdot \frac{a_x}{R}$

$\alpha = \frac{a_x}{R}$

$r = R$

line of action

(CH 9)

note $a_x = a_{xcm}$

TRANSLATION: $ma_x = mg \sin \theta - f_s$

ROTATION: $R \cdot f_s = I \frac{a_x}{R}$

$\rightarrow f_s = \frac{I a_x}{R^2}$

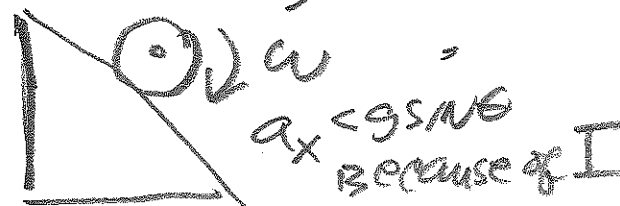
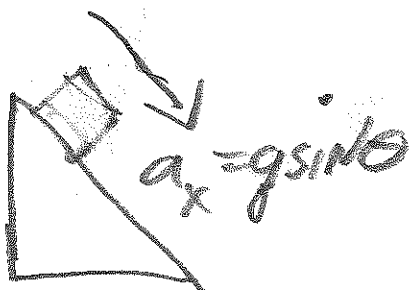
Solve for a_x :

ADD, cancel f_s

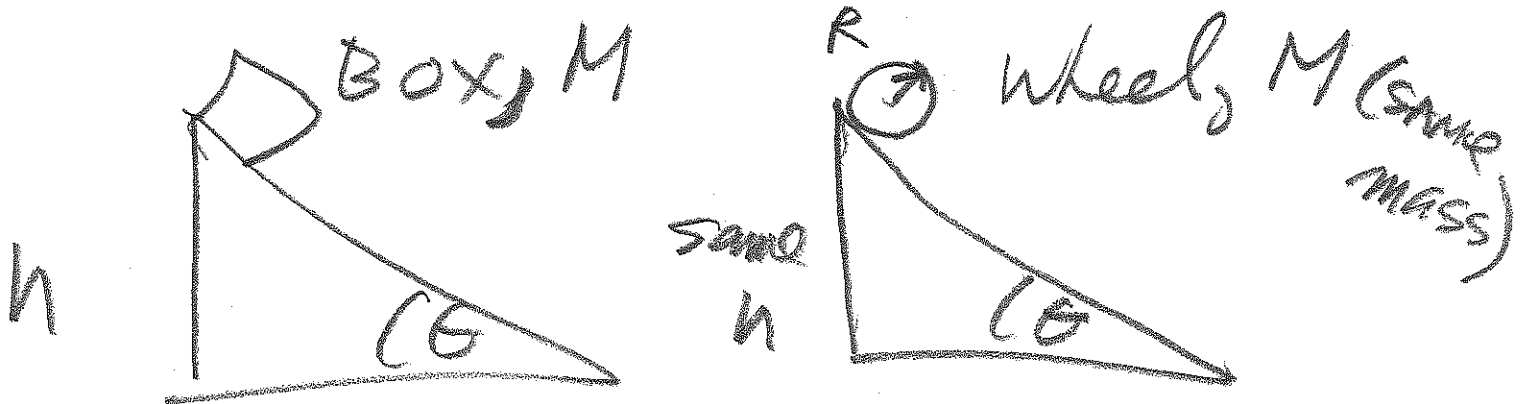
$$a_x \left(m + \frac{I}{R^2} \right) = mg \sin \theta$$

$$a_x = \frac{mg \sin \theta}{\left(m + \frac{I}{R^2} \right)} < g \sin \theta$$

COMPARE
SLIDING
 \rightarrow
BUT: NO FRICTION



Race between



no f_k

WHO WINS?

BOX!

$$g \sin \theta > \frac{M g \sin \theta}{\left(M + \frac{I}{R^2}\right)}$$

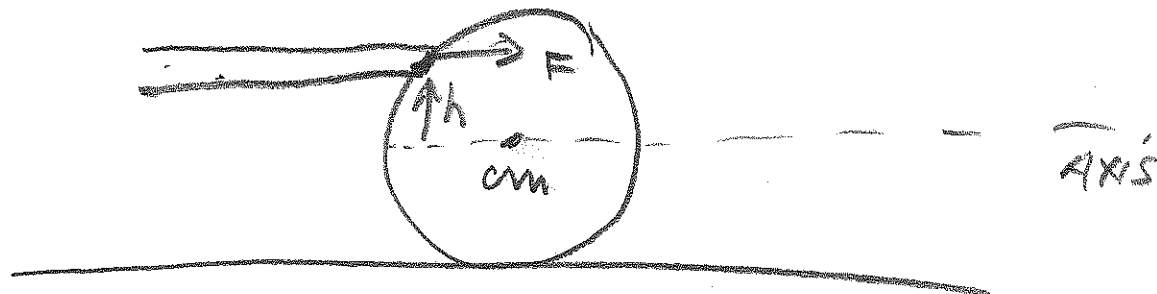
you can also use conservation of energy to see who wins: See

Example 10.5: $mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$

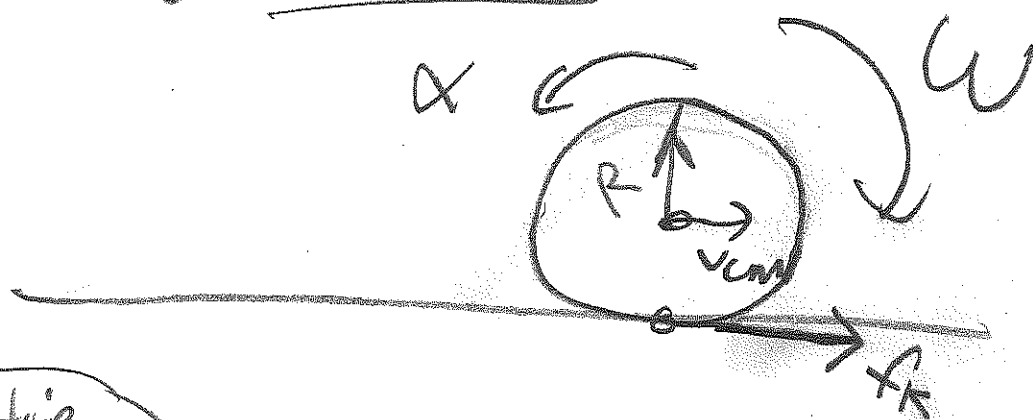
where $\omega = \frac{v}{R}$; FIND

v (at bottom.)

Example of Pool stick impulse on cue ball.



(ENGLISH) EXTRA SPIN



Define positive direction: \odot

$v_{cm} \perp \omega \cdot R$

ω decreases as CM moves right.

kinetic friction $\rightarrow f_k$ exerts a torque that reduces ω .

$I\alpha = -R \cdot f_k$; $\alpha = \frac{d\omega}{dt}$

Translation $\Sigma F_x = \text{pos} - \text{neg}$

$$\text{max} = f_k$$

$$m \frac{dv_{cm}}{dt} = f_k \Rightarrow v_{cm} = v_0 + \frac{f_k}{m} \cdot t$$

Rotation

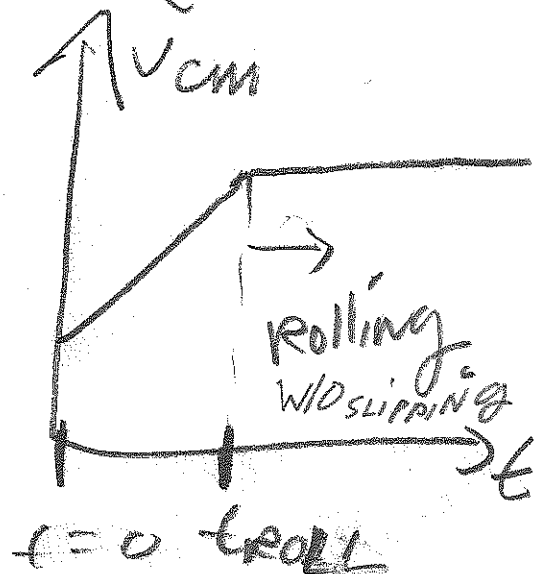
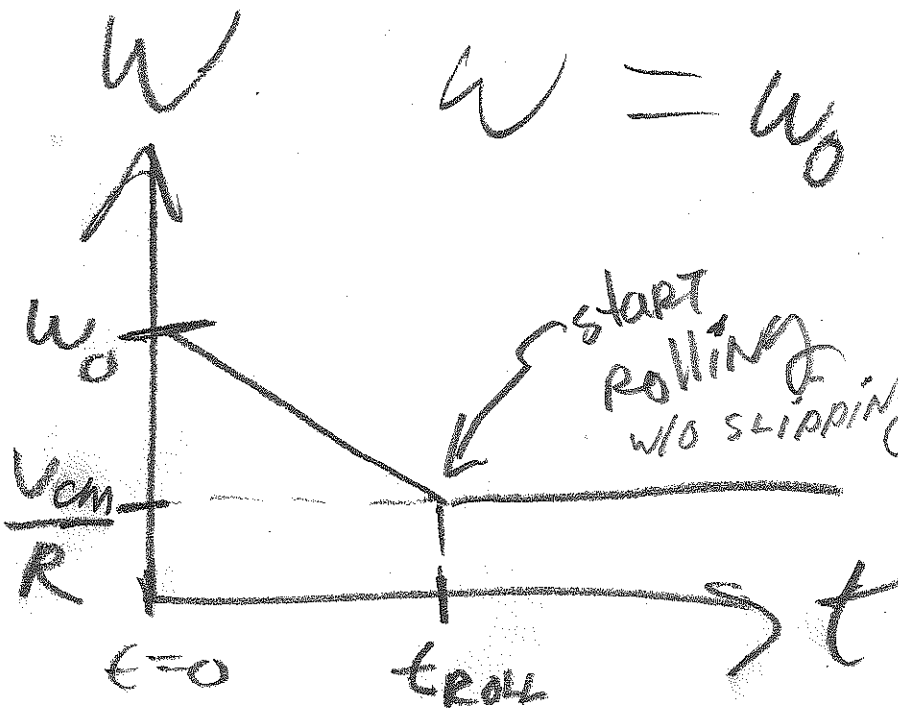
$$\tau = -R \cdot f_k$$

$$\tau \frac{d\omega}{dt} = -R \cdot f_k$$

$$\Rightarrow \omega = \omega_0 - \frac{R f_k}{I} t$$

$$\alpha = -\frac{R f_k}{I}$$

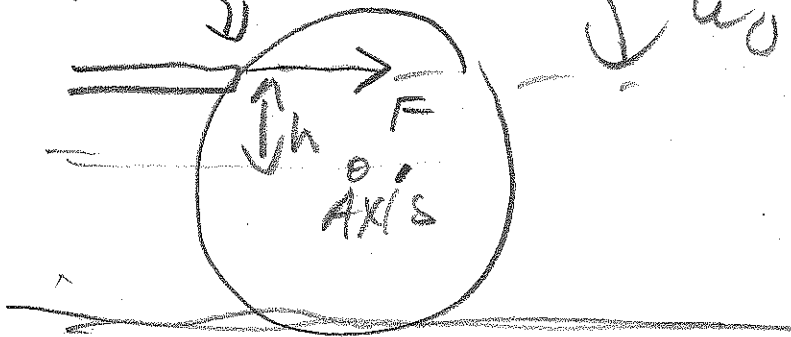
$$\omega = \omega_0 + \alpha \cdot t$$



$$v_{cm} = \omega \cdot R \Rightarrow \omega = \frac{v_{cm}}{R} \text{ (rolling)}$$

Z-AXIS \perp PAGE.

TIME OF CONTACT
 $= \Delta t$



$$\text{TORQUE} = \frac{\Delta L_z}{\Delta t}$$

$$h \cdot F = \frac{I(\omega_0 - 0)}{\Delta t}$$

Note:

$$\omega_0 = \frac{h F \Delta t}{I}$$

where $\Delta t =$ time
of impulse by
stick on ball.

The CM speeds up until $v_{cm} = \omega \cdot R$.

$$v_0 + \frac{f_k \Delta t}{m} = \left(\omega_0 - \frac{R f_k \Delta t}{I} \right) \cdot R$$

solve for $t = t_{roll}$ when
rolling w/o slipping
STARTS.

NOTE: $F = \frac{\Delta p_x}{\Delta t}$

$$F = \frac{mv_0 - 0}{\Delta t}$$

THUS:

$$v_0 = \frac{F \Delta t}{m}$$

MOMENT OF INERTIA PART 2 LAB

SEE EXAMPLE 10.3 OF TEXTBOOK

TODAY WE WILL VERIFY THE KINEMATIC LAW RELATING THE ANGLE OF ROTATION AND THE TIME. FROM PART 1, WE KNOW from EXAMPLE 10.3:

$h = \frac{1}{2}at^2$, where a is the linear acceleration of and h the vertical distance the hanging mass falls in a time t if it starts from rest. We also know the pulley's angular acceleration is $\alpha = a/r$, where r is the radius of the pulley axle drum. IN ADDITION WE KNOW $\theta = \frac{1}{2}\alpha t^2$ AND ALSO $\theta = h/r$. *Thus, we can we plot θ vs. t^2 with the expectation of a straight line whose slope is $\alpha/2$.*

FOR 3 GIVEN ROTATING SYSTEMS YOU WILL PLOT θ vs. t^2 for 4 values of the time t , picking the values of h that produce equally spaced value of t^2 . Here's how you do this. For a given value of maximum drop height h you will reduce the drop height 3 times so t^2 changes in equal increments. Suppose for example the maximum value of the height is H . Thus: $t_1 = t_{\max} = \sqrt{\frac{2H}{a}}$, here $H = h_1$. We want the next time to be such that $t_2^2 = \frac{3}{4}t_{\max}^2$, thus $h_2 = \frac{3}{4}H$; $t_3^2 = \frac{1}{2}t_{\max}^2$, thus $h_3 = \frac{1}{2}H$; $t_4^2 = \frac{1}{4}t_{\max}^2$, thus $h_4 = \frac{1}{4}H$.

YOU WILL PLOT THE ANGLE VS THE SQUARE OF TIME AND VERIFY THE LAW OF ROTATION BY OBSERVING A STRAIGHT LINE. YOU WILL MAKE THREE PLOTS OF

θ vs. t^2 : *With the spindle underneath*, YOU WILL PERFORM THE EXPERIMENT WITH (1) THE **DISK ALONE**, (2) THE **RING ALONE** AND THE (3) **RING PLUS THE DISK** AND COMPARE THE SLOPES OF THE LINES GIVEN BY θ vs. t^2 . You will get time t with a digital timer.

FOR EACH PLOT YOU WILL HAVE 4 DROP HEIGHTS AND FOR EACH OF THE 4 DROP HEIGHTS h , YOU WILL DROP THE HANGING MASS 4 TIMES AND COMPUTE THE AVERAGE TIME. For a particular drop height the angle is the height/ r , where r is the radius of the axle drum you will measure.

DATA SHEET:	
$r =$	
SPINDLE + RING	
$h_1 = H = \max h$	θ_1
TIME t	
a	
b	
c	
d	
Average t	
$h_2 = 3H/4$	θ_2
TIME t	
a	
b	
c	
d	
Average t	
$h_3 = H/2$	θ_3
TIME t	
a	
b	
c	
d	
Average t	
$h_4 = H/4$	θ_4
TIME t	
a	
b	
c	
d	
Average t	

SPINDLE + DISK	
$h_1 = H = \max h$	θ_1
TIME t	
a	
b	
c	
d	
Average t	
$h_2 = 3H/4$	θ_2
TIME t	
a	
b	
c	
d	
Average t	
$h_3 = H/2$	θ_3
TIME t	
a	
b	
c	
d	
Average t	
$h_4 = H/4$	θ_4
TIME t	
a	
b	
c	
d	
Average t	

(SPINDLE + DISK + RING)	θ_1
$h_1 = H = \max h$	
TIME t	
a	
b	
c	
d	
Average t	
$h_2 = 3H/4$	θ_2
TIME t	
a	
b	
c	
d	
Average t	
$h_3 = H/2$	θ_3
TIME t	
a	
b	
c	
d	
Average t	
$h_4 = H/4$	θ_4
TIME t	
a	
b	
c	
d	
Average t	

DO NOT DO TODAY unless

you have

time

FOR EACH SYSTEM, PLOT θ vs. t^2 . USE *LOGGER PRO* (WITH NO DEVICE CONNECTED) OR *EXCEL* TO

GRAPH THE LINE AND FIND THE SLOPE OF θ vs. t^2 .

Q1: EXPLAIN THE DIFFERENCES BETWEEN THE SLOPES OF THE 3 GRAPHS. DOES THE ORDERING OF THE SLOPE MAGNITUDES MAKE SENSE? WHICH SLOPE IS THE LARGEST? THE SMALLEST? INTERMEDIATE? EXPLAIN USING PHYSICAL PRINCIPLES AND EQUATIONS.

Q2: FROM YOUR SLOPE VALUE FOR THE SPINDLE + DISK, COMPUTE $I_{SP} + I_{DISK}$ AND COMPARE WITH THE VALUE OF $I_{SP} + I_{DISK}$ YOU OBTAINED IN PART 1 OF THIS LABORATORY.

Q2

$$\alpha = \frac{a}{r} = \frac{mg}{\left(\frac{I}{r} + mr\right)}$$

$$\alpha = \frac{mgr}{I + mr^2}$$

SPINDLE + DISK

COMPARED with $I_{DISK} + I_{SP}$ of PART 1 LAB

Solve for $I_{DISK} + I_{SP}$

SLOPE = $\frac{\alpha}{2}$

FIND α
FIND $I_{SP} + I_{DISK}$

$\theta = \frac{\alpha}{2} t^2$

t^2 t^2 t^2 t^2

GRAPH THE LINE AND FIND THE SLOPE OF θ vs. t^2 .

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SPINDLE + DISK

COMPARED with $I_{DISK} + I_{SP}$ of PART 1 LAB

Solve for $I_{DISK} + I_{SP}$

SLOPE = $\frac{\alpha}{2}$

FIND α
FIND $I_{SP} + I_{DISK}$

$\theta = \frac{\alpha}{2} t^2$