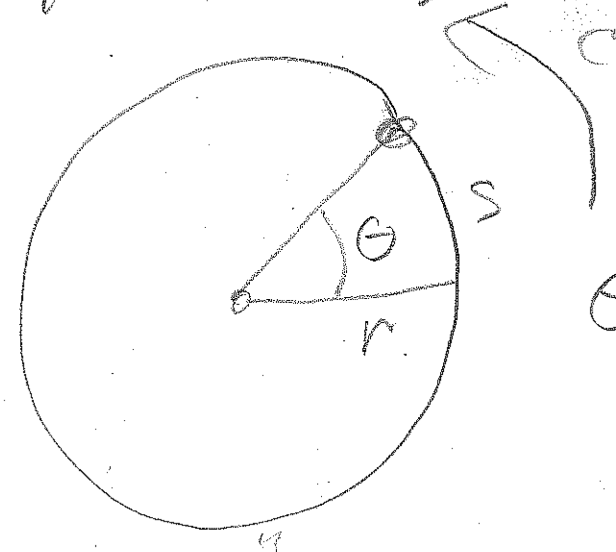
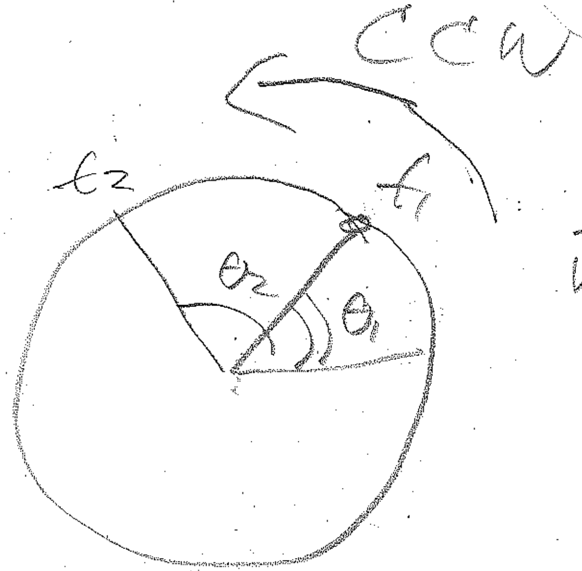
ch.9 4-15-13

conventions

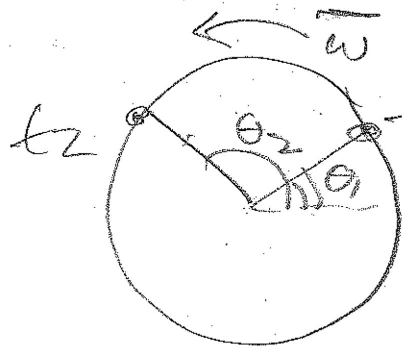


θ increasing

$$\theta = \frac{s}{r} \bmod 2\pi = \frac{C}{r} = \frac{2\pi r}{r} = 2\pi = 6.28\ldots \text{ RAD}$$



$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \left(\frac{\text{RAD}}{s}\right) = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

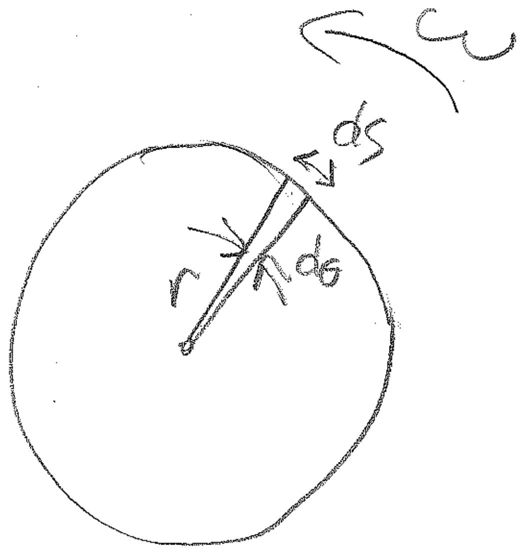


$\bar{\omega} > 0$ since CCW

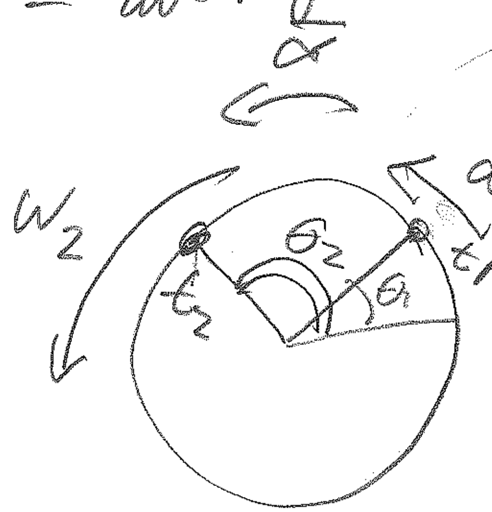
differential

limit!

$$\omega \equiv \frac{d\theta}{dt} \left(\frac{RAD}{S}\right)$$

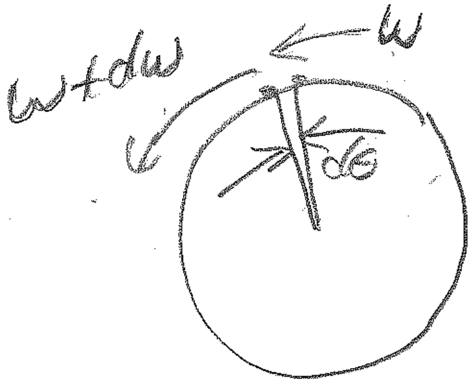


$\bar{\alpha}$ = average angular acceleration (RAD/s^2)



$$\bar{\alpha} \equiv \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_2}$$

$\bar{\alpha} > 0$ if $\omega_2 > \omega_1$

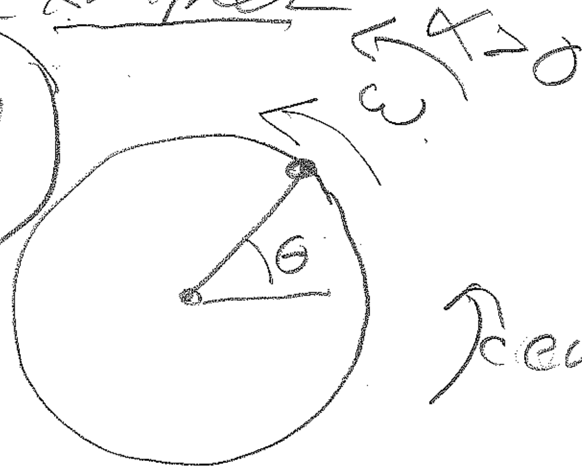


$$\alpha = \frac{(\omega + d\omega) - \omega}{(t + dt) - t}$$

$$\alpha = \frac{d\omega}{dt}$$

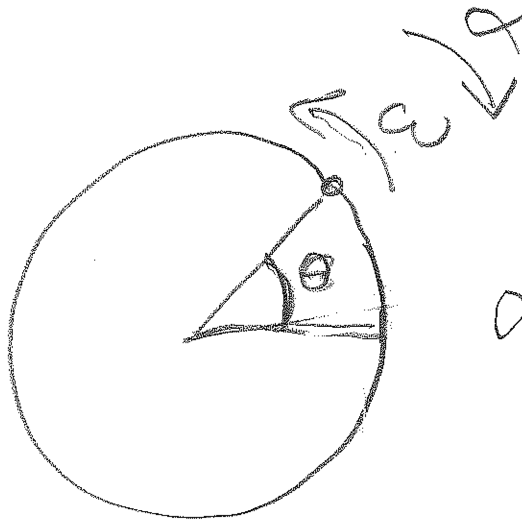
Examples

I. Speeding up



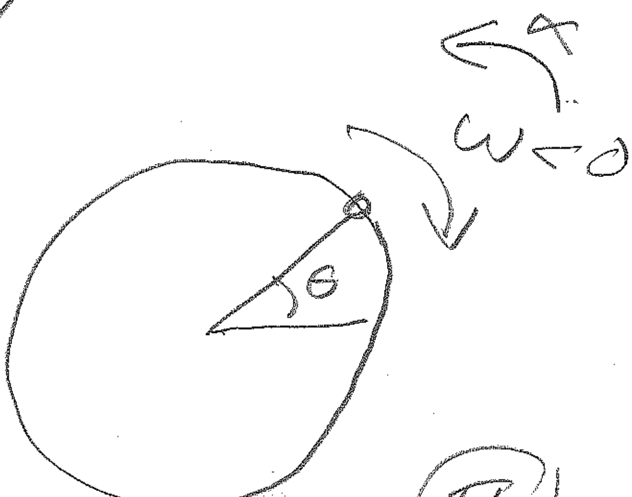
ccw ($\omega > 0$)

II. Slowing down

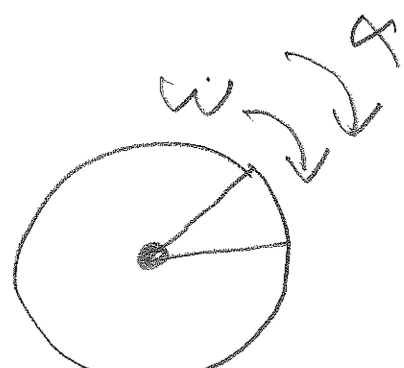


α cw

III. ALSO slowing DOWN

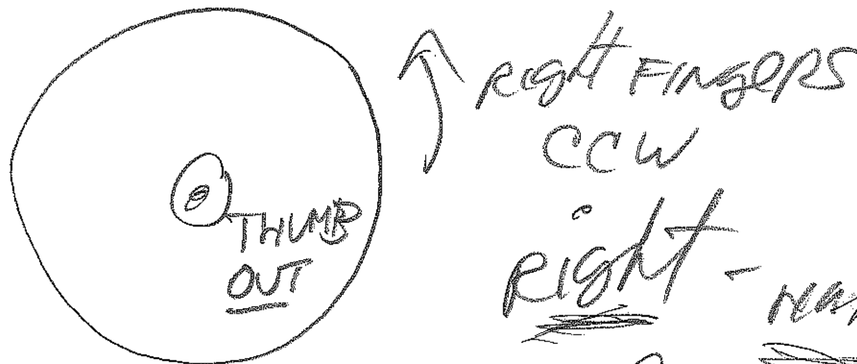


IV.



speeding up

anatomical basis for rotational conventions



⊙ OUT (positive)

⊗ IN (negative)

Right-hand-rule.

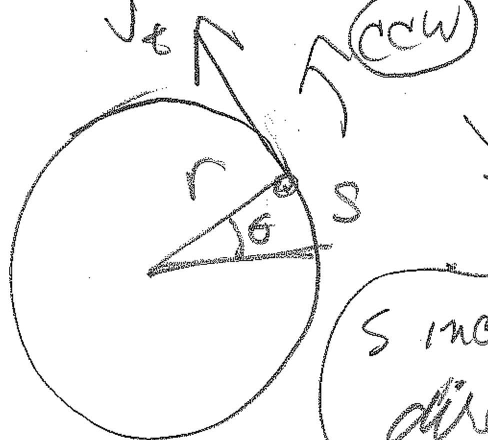
---

special case when α = constant

same equations as CH2:

CH2 → x (POS)	CH9 CCW (POS)
$v_{2x} = v_{1x} + a \cdot \Delta t$	$\omega_2 = \omega_1 + \alpha \cdot \Delta t$; $\Delta t = t_2 - t_1$
$\Delta x = v_1 \Delta t + \frac{1}{2} a \Delta t^2$	$\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha \Delta t^2$; $\Delta\theta = \theta_2 - \theta_1$
$\bar{v}_x = \frac{v_1 + v_2}{2}$	$\bar{\omega} = \frac{\omega_2 + \omega_1}{2}$ and $\Delta\theta = \bar{\omega} \cdot \Delta t$
and $v_2^2 = v_1^2 + 2 \cdot a \cdot \Delta x$	$\omega_2^2 = \omega_1^2 + 2 \cdot \alpha \cdot \Delta\theta$.

Relationship between linear and rotational quantities



$s = r\theta$

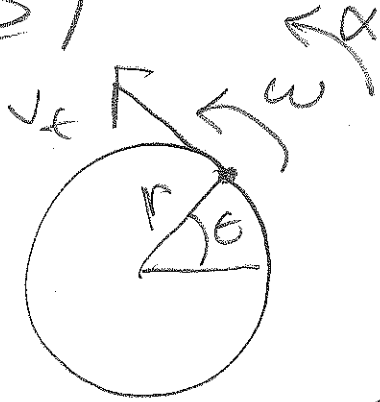
$v_t = r\dfrac{d\theta}{dt}$

$\boxed{v_t = r\omega}$

units $v_t\left(\dfrac{m}{s}\right)$

$v_t = \dfrac{ds}{dt}$ = tangential, linear velocity

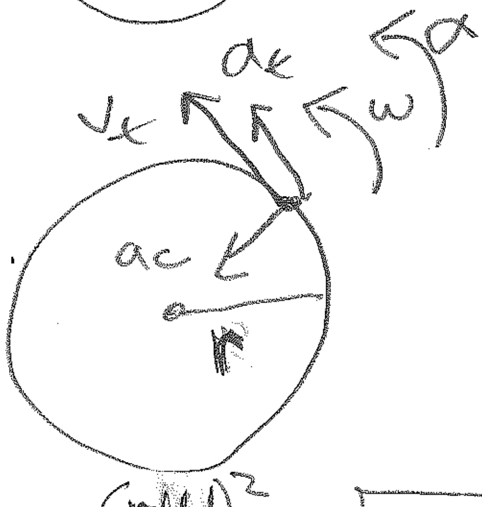
s increases in positive direction ↺



$a_t = \dfrac{d^2s}{dt^2}$

$s = r\theta$

$\dfrac{d^2s}{dt^2} = r\dfrac{d^2\theta}{dt^2}$

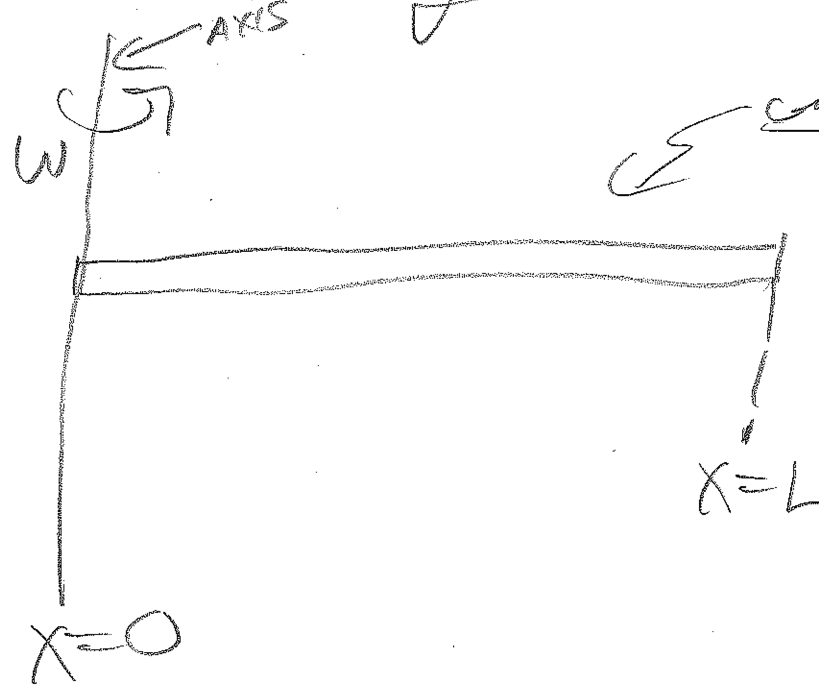


$a_t = r\alpha$ $\left(\dfrac{m}{s^2}\right)$

$\left(\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}\right.$ since $\left.\omega = \dfrac{d\theta}{dt}\right)$

$a_c = \dfrac{v_t^2}{r} = \dfrac{(r\omega)^2}{r} = \boxed{r\omega^2} \rightarrow a_c = r\omega^2$ $\left(\dfrac{m}{s^2}\right)$

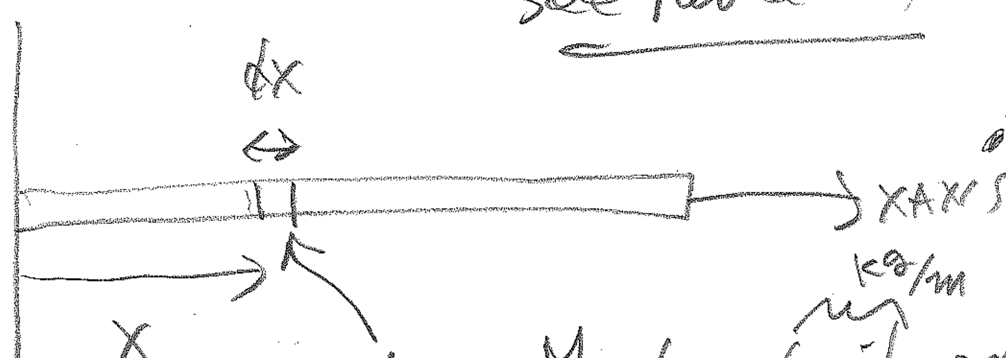
Derivation of I for a Rod:



UNIFORM ROD of length L, TOTAL mass M.

Find I about X through end.

see table 9.2



SUM OVER BODY

$$I = \sum_{i=1}^{N} m_i r_i^2$$

$$dm = \frac{M}{L} \cdot dx = \underbrace{\text{density}}_{\text{kg/m}} \cdot dx$$

$$dI = dm \cdot x^2 \rightarrow \int_0^L \frac{M}{L} dx \cdot x^2 = \frac{M}{L} \frac{x^3}{3}\Big|_0^L = \frac{ML^2}{3}$$

$$I = \int_0^L dm \cdot x^2$$ ← infinite sum

Parallel-Axis Theorem

Section 9.5

$$I_{AXIS} = I_{cm} + M \cdot D^2$$

D = distance between center of mass (cm) and new AXIS.

FLASH BACK:

Center of mass review CH 8:

N particles

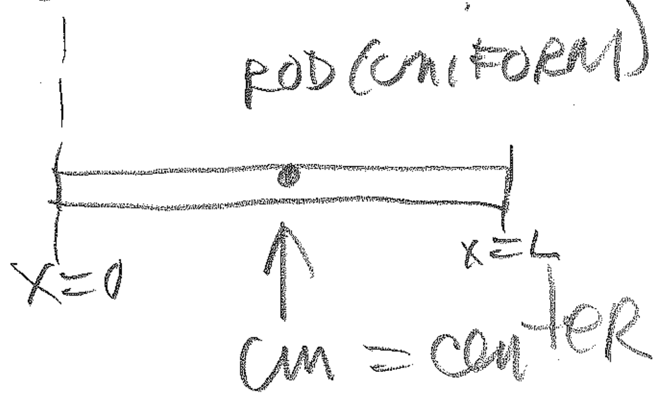
$$X_{cm} = \frac{\sum_{i=1}^{N} m_i x_i}{\sum m_i} \rightarrow X_{cm} = \frac{\int dm\, x}{\int dm}$$

discreet SUM

solid Body

CM REVIEW

Uniform object: generally, cm at center.

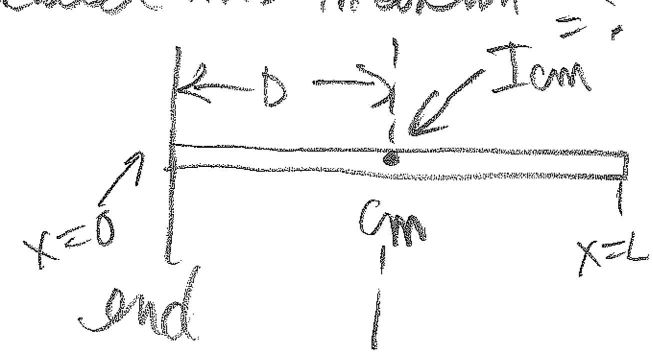


$$X_{cm} = \frac{\int_0^L dm \cdot x}{\int dm} = \frac{\int_0^L \frac{M}{L} \cdot dx \cdot x}{\int_0^L dm} = \frac{\frac{M}{L}\frac{x^2}{2}\Big|_0^L}{M} = \frac{L}{2}$$

$$dm = \frac{M}{L} \cdot dx$$

Parallel axis theorem = ?

$$I_{end} = \frac{ML^2}{3}$$



$$I_{end} = I_{cm} + M\left(\frac{L}{2}\right)^2$$

$$\frac{ML^2}{3} = I_{cm} + \frac{ML^2}{4}$$

$$\frac{ML^2}{3} = I_{cm} + \frac{ML^2}{4}$$

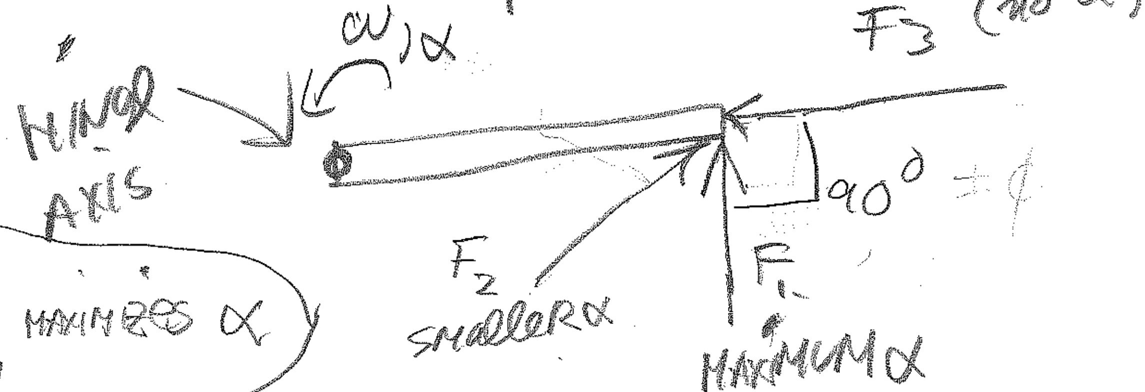
$$\frac{ML^2}{3} - \frac{ML^2}{4} = I_{cm}$$

$$\frac{4}{12}ML^2 - \frac{3}{12}\cdot ML^2 = I_{cm}$$

$$\frac{ML^2}{12} = I_{cm} \text{ (See table 9.2)}$$

TORQUE CH10

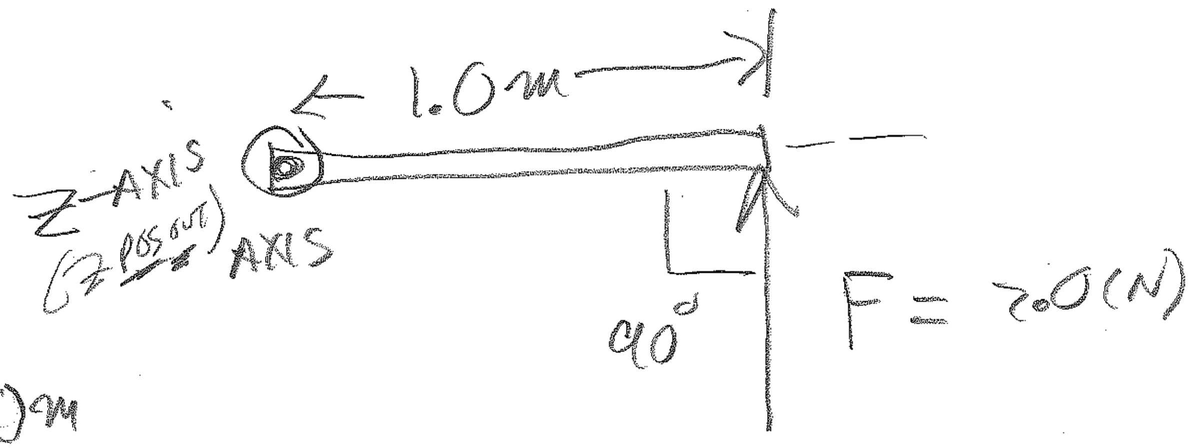
top view of door



$\vec{F}_1$ MAXIMIZES α

TORQUE $= I \cdot \alpha$

newton's 2ND LAW of rotation



$L = 1.0\,m$

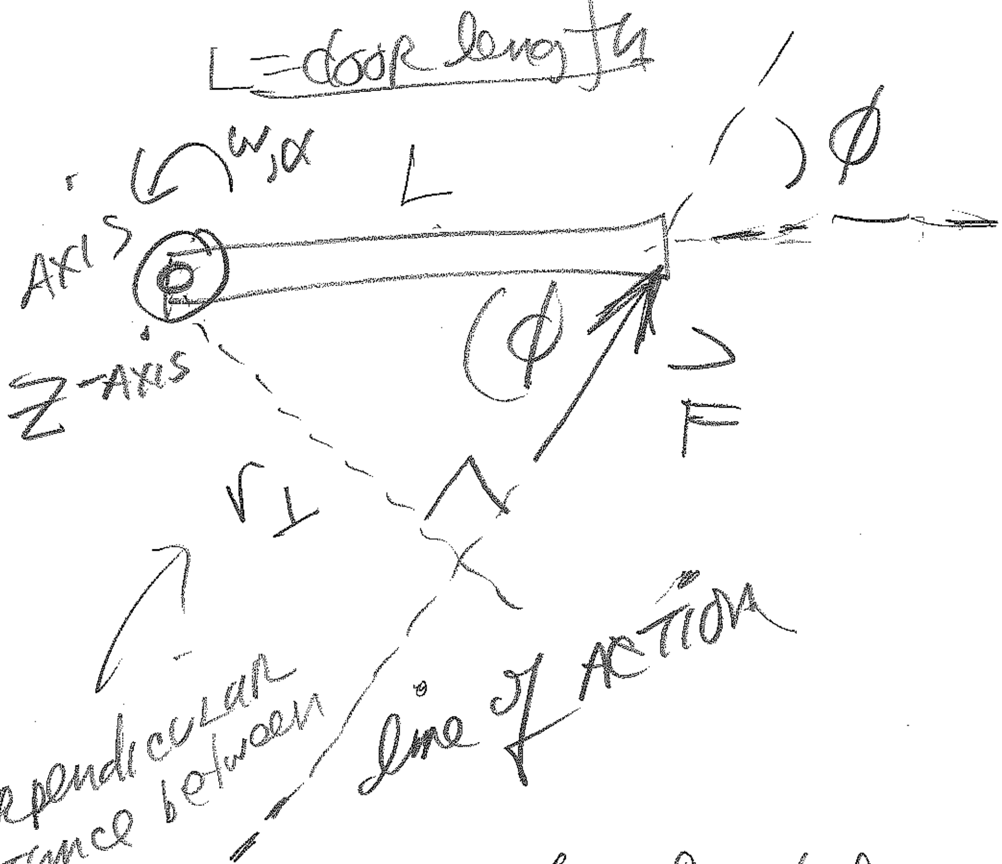
TORQUE $= \tau_z = I \cdot \alpha$

note: IF $\alpha > 0$ α

$\tau_z > 0$

note: I is about AXIS (Z-AXIS)

WHAT is TORQUE?



perpendicular distance between AXIS and line of action of force

$r_\perp$ is found from geometry:

$$|\tau_z| = r_\perp \cdot F,$$

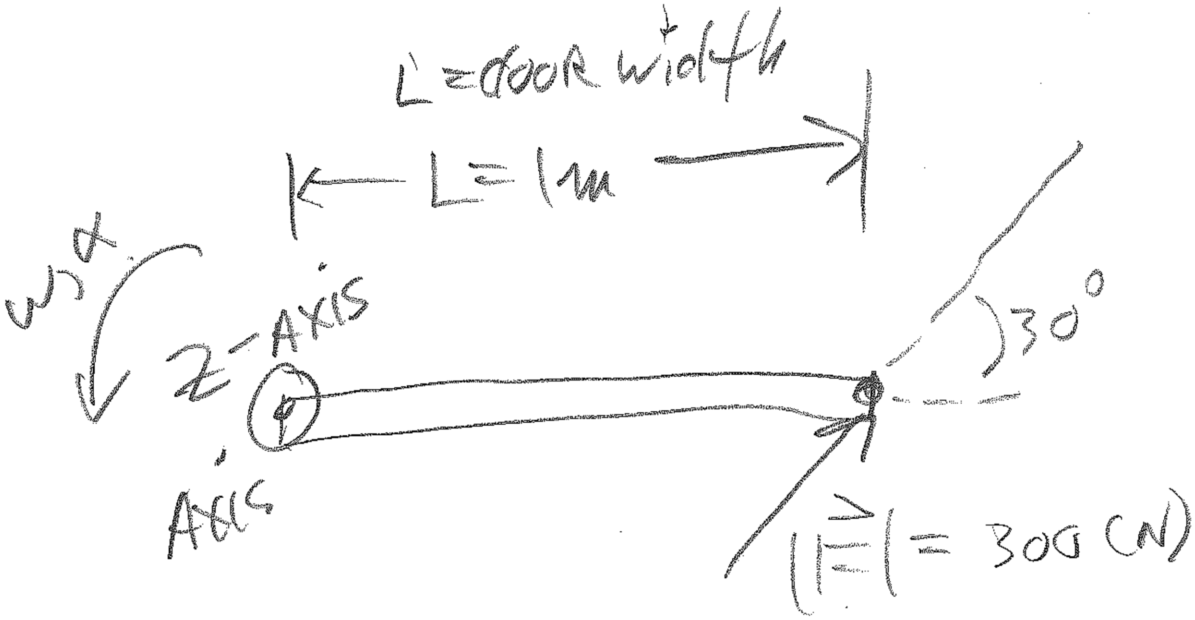
$F = |\vec{F}|$

Example: $r_\perp = L \cdot \sin\phi$

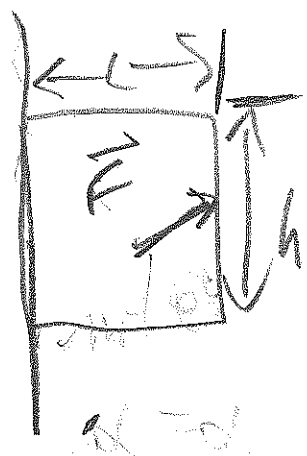
$$|\tau_z| = (L \cdot \sin\phi) \cdot F$$

NOTE: $|\tau_z|$ = MAXIMUM when $\phi = 90^\circ$

Note: $\tau_z > 0$ if rotation is ↺ after starting from rest.



$$M = 25.0\,\text{kg} \approx 50\ \text{lbs}$$

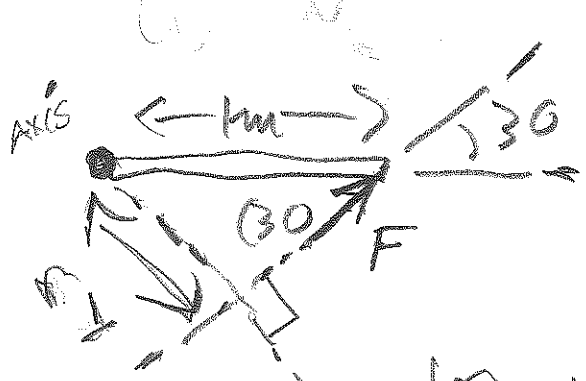


what is α?

$$I\alpha = \tau_z$$

$$I = ?$$

$$I = \frac{ML^2}{3} = \frac{(25.0\,\text{kg})(1\,\text{m})^2}{3} = \frac{25}{3}\,\text{kg}\cdot\text{m}^2$$



$$\alpha_z = \frac{\tau_z}{I} = \alpha$$

$$|\tau_z| = r_\perp \cdot F = (1\,\text{m})\sin 30^\circ \cdot 300\ \text{N}\cdot\text{m}$$
$$= 150\ \text{N}\cdot\text{m}$$

$$\alpha_z = \frac{150\ N\cdot m}{\frac{1}{3}ML^2} = \alpha$$

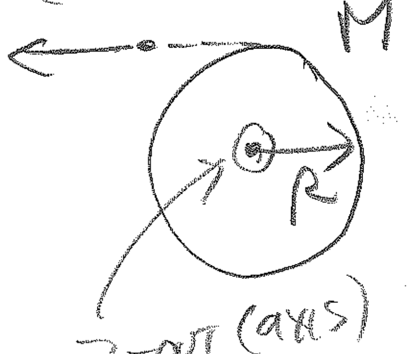
$$\alpha = \alpha_z = \frac{150\ N\cdot m}{\frac{1}{3}(25.0)kg\cdot(1m)^2} = 18\ \frac{RAD}{s^2}$$

units check

$$\boxed{\begin{array}{l} N = \dfrac{kg\ m}{s^2} \\ \underline{\text{THUS:}} \\ \dfrac{\frac{kg\ m}{s^2}\cdot m}{kg\cdot m^2} = \dfrac{RAD}{s^2} \end{array}}$$

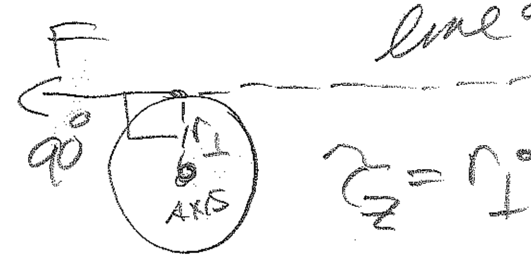
Example 10.2
" 10.3

Ex 10.2
9.0(N)



2 $\alpha = ?$

line of action



$\tau_z = r_\perp \cdot F$

ASSUME: CYLINDER

$\Rightarrow I = \frac{MR^2}{2}$

(table 9.2)

$r_\perp = R$

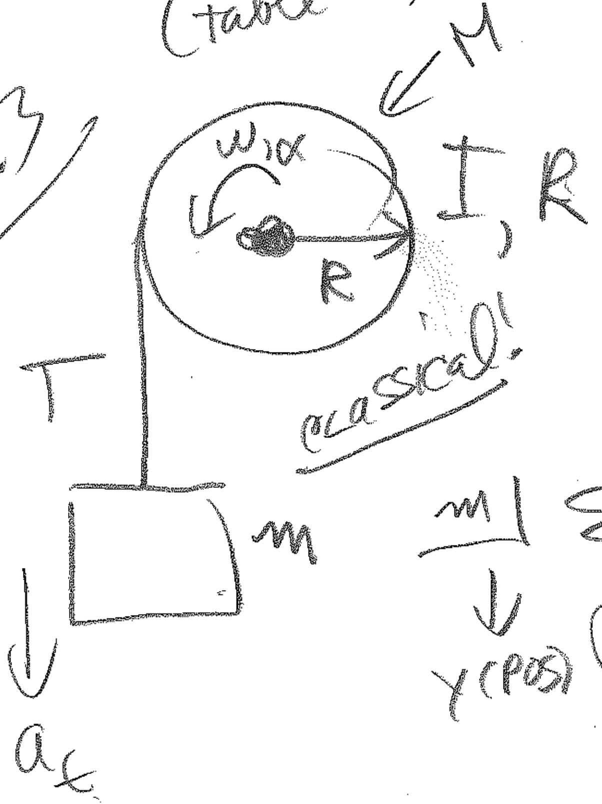
$I\alpha = R \cdot F$

$\frac{1}{2} MR^2 \cdot \alpha = R \cdot F$

$\alpha = \frac{2F}{MR}$

NOTE: α decreases as M OR R increase

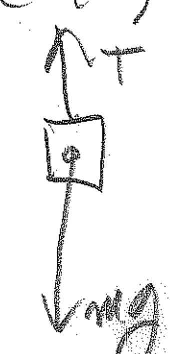
Ex 10.3



classical!

10.3 | ISOLATE EACH OBJECT

NOTE: $\alpha = \frac{a_t}{R}$ (CW a)



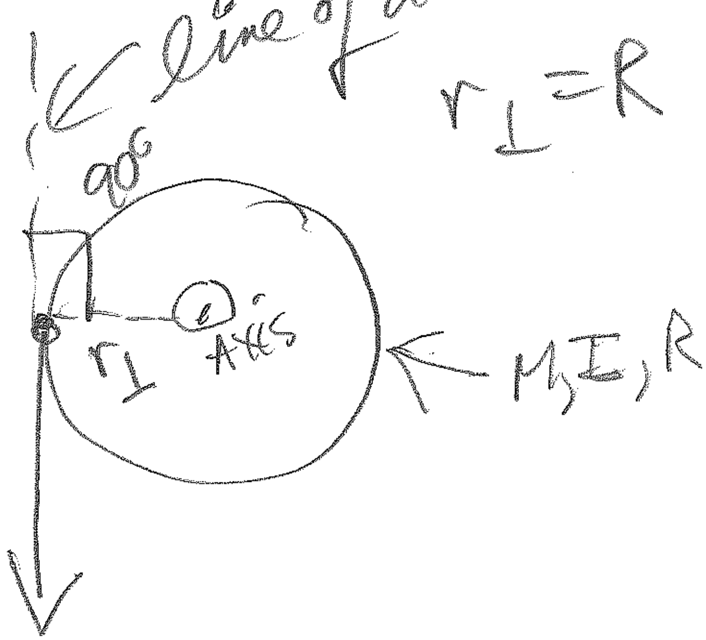
m | ΣF_y = pos − neg

↓ y (pos)

$ma_t = mg - T$

(I)

PULLEY, M, R



$$I\alpha = \tau_z$$

$$I\frac{a_t}{R} = r_\perp \cdot T$$

T

$$I\frac{a_t}{R} = R \cdot T$$

$$I\frac{a_t}{R^2} = T \quad \text{II}$$

Solve for a_t:

ADD I + II to cancel T:

$$I \cdot \frac{a_t}{R^2} + m a_t = mg$$

$I = \frac{1}{2}MR^2$ $\quad a_t = \dfrac{mg}{\left(m + \frac{I}{R^2}\right)} = \dfrac{mg}{\left(m + \frac{M}{2}\right)}$

$a_t < g$ because of M