## Capacitors

## 24

The charge $q$ on a capacitor's plate is proportional to the potential difference $V$ across the capacitor. We express this relationship with

$$
V=\frac{q}{c}
$$

where C is a proportionality constant known as the capacitance. C is measured in the unit of the farad, F, ( 1 farad $=1$ coulomb/volt).

If a capacitor of capacitance C (in farads), initially charged to a potential $V_{0}$ (volts) is comected across a resistor $R$ (in ohms), a time-dependent current will flow according to Olm's law. This situation is shown by the RC (resistor-capacitor) circuit below when the switch is connecting terminals 33 and 34 .


Figutre 1
As the charge flows, the charge $q$ on the capacitor is depleted, reducing the potential across the capacitor, which in turn reduces the current. This process creates an exponentially decreasing current, modeled by

$$
V(t)=V_{0} e^{-\frac{t}{R C}}
$$

The rate of the decrease is determined by the product $R \mathrm{C}$, known as the time constant of the circuit. A large time constant means that the capacitor will discharge slowly.

In contrast, when the capacitor is charged, the potential across it approaches the final value exponentially, modeled by

$$
V(t)=V_{0}\left(1-e^{-\frac{t}{R C}}\right)
$$

The same time constant, $R \mathrm{C}$, describes the rate of charging as well as discharging.

## OBJECTIVES

- Measure an experimental time constant of a resistor-capacitor circuit.
- Compare the time constant to the value predicted from the component values of the resistance and capacitance.
- Measure the potential across a capacitor as a function of time as it discharges and as it charges.
- Fit an exponential function to the data. One of the fit parameters corresponds to an experimental time constant.


## MATERIALS

computer
Vernier computer interface Logger Pro
Vernier Differential Voltage Probe connecting wires with clips

Vernier Circuit Board with batteries or $10 \mu \mathrm{~F}$ non-polarized capacitor $100 \mathrm{k} \Omega$ and $47 \mathrm{k} \Omega$ resistors two C - or D-cell batteries with holder single-pole, double-throw switch

## PRELIMINARY QUESTIONS

1. Consider a candy jar, initially with 1000 candies. You walk past it once each hour. Since you do not want anyone to notice that you are taking candy, each time you take only $10 \%$ of the candies remaining in the jar. Sketch a graph of the number of candies for a few hours.
2. How would the graph change if instead of removing $10 \%$ of the candies, you removed $20 \%$ ? Sketch your new graph.

## PROCEDURE

1. Set up the equipment.
a. Connect the circuit using the $10 \mu \mathrm{~F}$ capacitor and the $100 \mathrm{k} \Omega$ resistor, as shown in Figure 1. Note: The numbers in the figure refer to the numbered terminals on the Vernier Circuit Board.
b. Record the values of your resistor and capacitor in your data table, as well as any tolerance values marked on them.
c. Connect the Differential Voltage Probe to the computer interface.
d. Connect the clip leads on the Differential Voltage Probe across the capacitor. Note: Connect the red lead to the side of the capacitor connected to the resistor. Connect the black lead to the other side of the capacitor.
e. Set Switch 1, SW1, located below the battery holder on the Vernier Circuit Board, to 3.0 V .
2. Open the file in the " 24 Capacitors" file in the Physics with Vernier folder.
3. Set Switch 2, SW2, to charge the capacitor for 10 seconds (so the switch is closer to terminal 32). Watch the voltage reading to see if the potential is still increasing.
4. Click Collect to begin data collection. As soon as graphing starts, flip Switch 2 to discharge the capacitor. Your data shows a constant value initially, then a decreasing function.
5. To compare your data to the model, select only the data after the potential has started to decrease by clicking and dragging across the curved portion of the graph; that is, omit the constant portions on either end of the discharge cycle. Click Curve Fit, 周, and from the function selection box, choose the Natural Exponent function, $A * \exp (-\mathrm{C} t)+B$. Click Try fit , and inspect the fit. Click or .
6. Record the value of the fit parameters in your data table. Notice that the C used in the curve fit is not the same as the C representing capacitance. Compare the fit equation to the mathematical model for a capacitor discharge proposed in the introduction.

$$
V(t)=V_{0} e^{-\frac{t}{R C}}
$$

How is fit constant $C$ related to the time constant of the circuit, which was defined in the introduction?
7. Print or sketch the graph of potential vs. time. Choose Store Latest Run from the Experiment menu to store your data. You will need the data for later analysis.
8. The capacitor is now discharged. To monitor the charging process, click collect. As soon as data collection begins, change Switch 2 so the capacitor charges. Allow data collection to run to completion.
9. This time you will compare your data to the mathematical model for a capacitor charging,

$$
V(t)=V_{0}\left(1-e^{-\frac{t}{R C}}\right)
$$

Select the data beginning after the potential has started to increase, omitting portions on either end of the charge cycle. Click Curve Fit, 忍, and from the function selection box, choose the Inverse Exponent function, $A *(1-\exp (-\mathrm{C} t))+B$. Check the time offset curve fit box. Click iny fit and inspect the fit. Click ok to return to the main graph.
10. Record the value of the fit parameters in your data table. Compare the fit equation to the mathematical model for a charging capacitor.
11. Hide your first runs by choosing Hide Data Set from the Data menu. Remove any remaining fit information by clicking the upper left corner in the floating boxes.
12. Now you will repeat the experiment with a resistor of lower value. How do you think this change will affect the way the capacitor discharges? Rebuild your circuit using the $47 \mathrm{k} \Omega$ resistor and repeat Steps 3-10.

## DATA TABLE

|  | Fit parameters |  |  |  |  | Resistor | Capacitor |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time <br> Trial <br> Constant |  |  |  |  |  |  |  |
| Discharge 1 |  | B | C | $1 / \mathrm{C}$ | R <br> $(\Omega)$ | C <br> (F) | RC <br> (s) |
| Charge 1 |  |  |  |  |  |  |  |
| Discharge 2 |  |  |  |  |  |  |  |
| Charge 2 |  |  |  |  |  |  |  |

## ANALYSIS

1. In the data table, calculate the time constant of the circuit used; that is, the product of resistance in ohms and capacitance in farads. Note: $1 \Omega \mathrm{~F}=1 \mathrm{~s}$
2. Calculate and enter in the data table the inverse of the fit constant $C$ for each trial. Now compare each of these values to the time constant of your circuit. How is the fit parameter $A$ related to your experiment?
3. Resistors and capacitors are not marked with their exact values, but only approximate values with a tolerance. Determine the tolerance marked on the resistors and capacitors you are using. If there is a discrepancy between the two quantities compared in Question 2, can the tolerance values explain the difference?
4. What was the effect of reducing the resistance of the resistor on the way the capacitor discharged?
5. How would the graphs of your discharge graph look if you plotted the natural logarithm of the potential across the capacitor vs. time? Sketch a prediction. Show Run 1 (the first discharge of the capacitor) and hide the remaining runs. Click the $y$-axis label Select $\ln (V)$.
6. What is the significance of the slope of the plot of $\ln (V) v s$. time for a capacitor discharge circuit?

## EXTENSIONS

1. What fraction of the initial potential remains after one time constant has passed? After two time constants? Three?
2. Instead of a resistor, use a small flashlight bulb. To light the bulb for a perceptible time, use a large capacitor (approximately 1 F ). Collect data. Explain the shape of the graph.
3. Try different value resistors and capacitors and see how the capacitor discharge curves change.
4. Try two $10 \mu \mathrm{~F}$ capacitors in parallel. Predict what happens to the time constant. Repeat the discharge measurement and determine the time constant of the new circuit using a curve fit.
5. Try two $10 \mu \mathrm{~F}$ capacitors in series. Predict what will happen to the time constant. Repeat the discharge measurement and determine the time constant for the new circuit using a curve fit.

# Vernier Lab Safety Instructions Disclaimer 

## THIS IS AN EVALUATION COPY OF THE VERNIER STUDENT LAB.

This copy does not include:

- Safety information
- Essential instructor background information
- Directions for preparing solutions
- Important tips for successfully doing these labs

The complete Physics with Vernier lab manual includes 35 labs and essential teacher information. The full lab book is available for purchase at: www.vernier.com/pwv


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