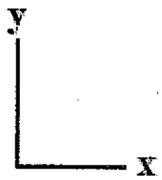
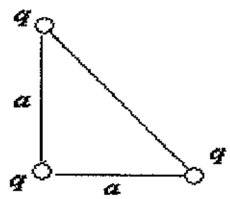
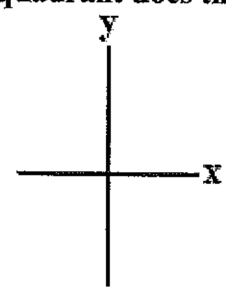
Scroll down for solutions

1. (40 points) Three point charges are arranged in a right-angled isosceles triangle as in the figure below. The angle between the two short sides is 90 degrees. The two short sides of the triangle have length a. All charges have the same value q. Most of this problem uses symbols---unless otherwise noted, *use a, q and other symbols as needed.* You will be investigating the *net force* on the charge in the *lower right corner* due to the other two charges. As needed, assume a right-handed coordinate system shown below:

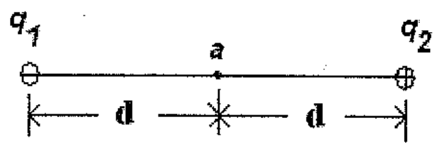




(a) (8 points) What is the x- component F_{xnet} of the *net* force on the charge in the lower right corner due to the other two charges?
(b) (8 points) What is the y- component F_{ynet} of the *net* force on the charge in the lower right corner due to the other two charges?
(c) (8 points) What is the angle the *net* force makes with the positive x-axis. Note: In this case, your answer will be a number. Give your answer in degrees.
(d) (8 points) What is the *magnitude* of the net force on the charge in the lower right corner due to the other charges?
(e) (3 points) Assume $a = 1.00$ m and $q = 2.00 \times 10^{-6}$ C. Plug in these numbers to evaluate the *magnitude* of the net force on the charge in the lower right corner due to the other charges?
(f) (5 points) **Make a sketch of the net force vector on the x-y axes below with the tail of the arrow at the origin. Label the angle with the x-axis. What quadrant does the vector lie in? Your picture should indicate the correct quadrant.**



2. (40 points) THE ELECTRIC FIELD OF AN ELECTRIC DIPOLE. This is an approximate model of a water molecule. Point charges q_1 and q_2 of $-1.00\text{x}10^{-19}$ C and $1.00\text{x}10^{-19}$ C, respectively, are placed $0.600\text{ x}10^{-12}$ m apart. Thus $d = 0.300\text{ x}10^{-12}$ m in fig. 1 below. This combination of two charges with equal magnitude and opposite sign is called an *electric dipole.* **Use *symbols* until the last steps when you plug in values to get numerical answers.**



fig, 1

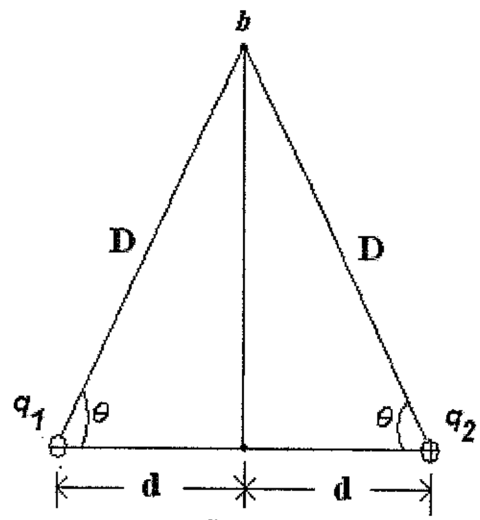
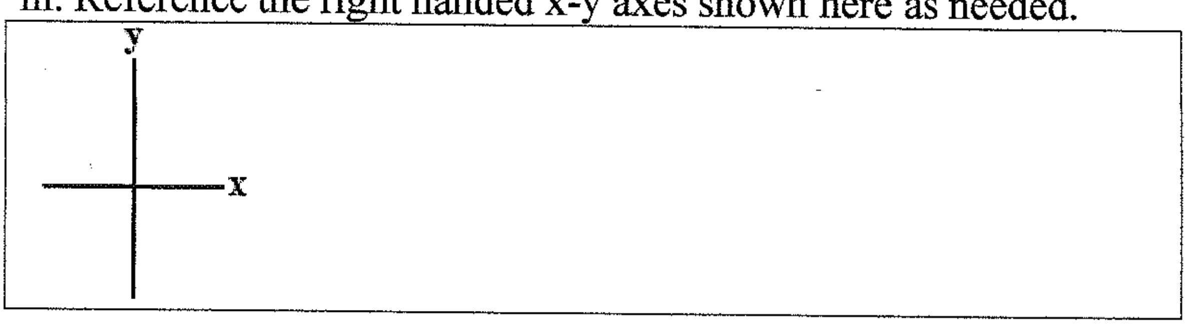


fig. 2

(a) (8 points) Refer to fig. 1 above. What must be the *direction* of the net electric field at point *a* midway between the two charges? Indicate this direction by drawing an arrow in fig. 1 with its tail at point *a*.

(b) (8 points) Refer to fig. 1 above. What is the *magnitude* E_{net} of the *net* electric field at point *a* midway between the two charges?

For the next parts, next page, see fig. 2. Point *b* is on the perpendicular bisector of the line connecting the two charges. Let the distance between either charge and point *b* be $D = 0.600\text{ x}10^{-12}$ m. Reference the right handed x-y axes shown here as needed.



(c) (8 points) Use basic trigonometry to find the ***value*** of angle θ shown in fig. 2.

(d) (6 points) At point *b*, what must be the y-component E_{ynet} of the *net* electric field due to the two charges? Show work.

(e) (6 points) At point *b*, what is the x-component E_{xnet} of the *net* electric field due to the two charges?

(f) (2 points) At point *b*, what is the direction of the *net* electric field due to the two charges? Indicate this direction by drawing an arrow in fig. 2 with its tail at point *b*.

(g) (2 points) At point *b*, what is the magnitude of the *net* electric field due to the two charges? What is the relationship between your answer here and your answer to part (e)?

3. (40 points) Two protons are held fixed (i.e. , do not move) 10.0 cm apart. Refer to fig. 1. Thus the distance d shown is 0.0500 m. Point *a* is on the perpendicular bisector of the line connecting the two protons. Also, point *a* is a distance h = 12.00 cm = 0.1200 m above the line connecting the two protons. The proton charge is *e* = 1.6×10^{-19} C. **Use *symbols* until the last steps when you plug in values to get numerical answers.**

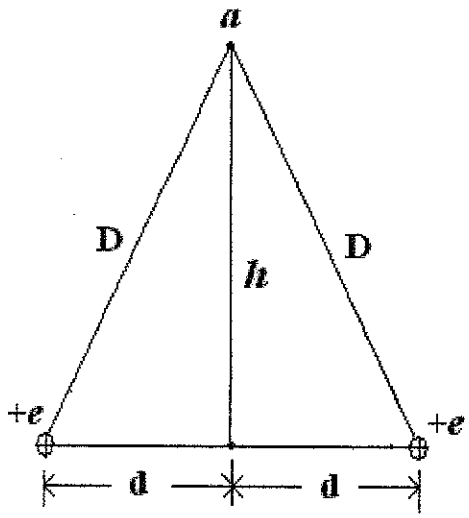


fig. 1

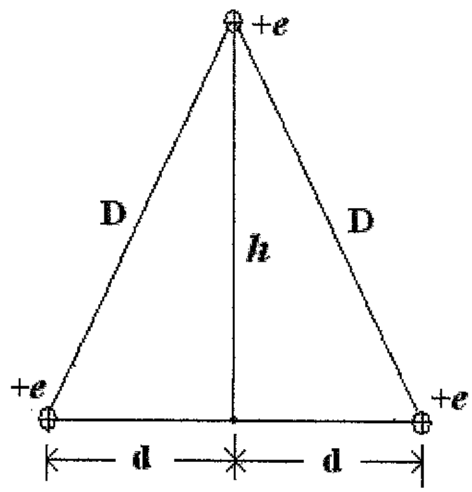


fig. 2

(a) (10 points) See fig. 1. Use the Pythagorean Theorem to find the distance D between either proton and point *a*. Use symbols until the very last step when you plug in numbers to get your numerical answer.

(b) (15 points) See fig. 1. Calculate the electric potential at point *a* due to the two protons. Try to use symbols until the very last step when you plug in numbers to get your numerical answer.

(c) (15 points) See fig. 2. A *third* proton is released from rest at point *a*. (The other two protons remain at rest.) What is the speed *v* of this third proton when it very far (i.e., at an infinite distance) from other two protons? The proton mass is 1.67×10^{-27}g.

4. (22 points) In figure 1 below each capacitor has $C = 4.00\ \mu F$. and the battery maintains a voltage difference between points a and b given by $V_{ab} = 100.00$ (V). In figure 2 below we show the three capacitors replaced by an equivalent single capacitor C_{eq} and the *same* voltage V_{ab} applied between points a and b.

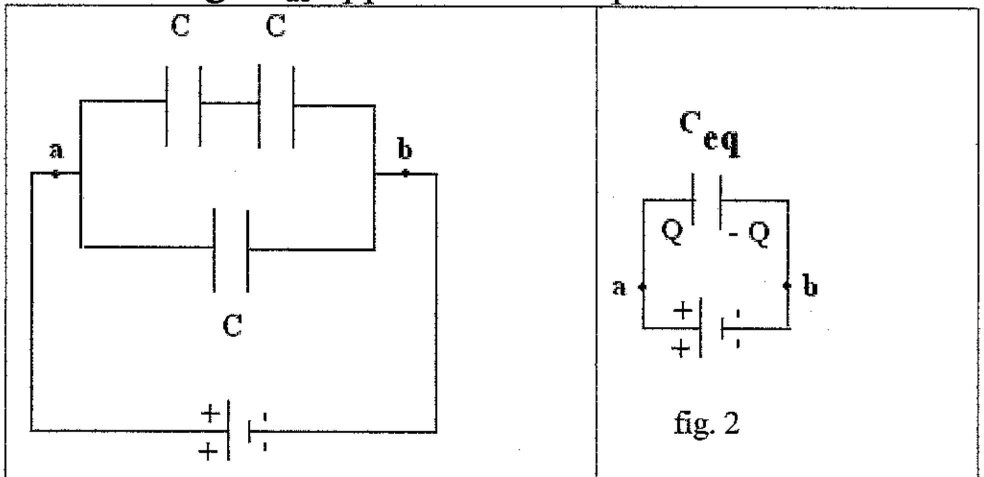


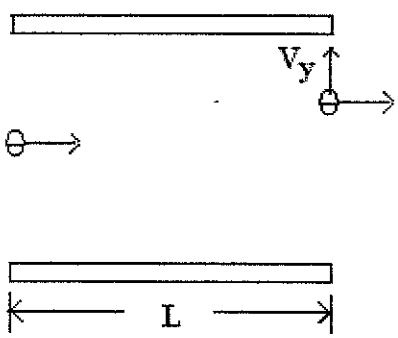
fig. 1 fig. 2

(a) (10 points) What is the value of the equivalent capacitor C_{eq} ?

(b) (8 points) In fig. 2 , what is the value of the charge Q associated with the equivalent capacitor C_{eq} ?

(c) (4 points) What is the value of the charge on the positive plate of each of the three capacitors? Refer to fig. 1 and fig. 2.

5. (8 points) An electron moves initially to the right as it enters a region between two parallel plates that have charges of equal magnitude and opposite sign. The *initial* horizontally directed speed is 6.00×10^{6} m/s as shown by the arrow at the left end of the plates. The uniform electric field between the plates has magnitude E = 12.00 N/C. The time the electron spends between the plates is t = 0.500 μs. When the electron finally exits the region between the plates, the electron has picked up a y-component of velocity V_y shown by the vertical arrow at the right end of the plates. Let the positive y-direction be upward and the positive x-direction be rightward.



(a) (2 points) Which plate is positive, the bottom or the top one? Explain.
(b) (2 points) What is the direction of the electric field between the plates up or down? Explain.
(c) (4 points) What is the value of V_y ? The electron mass is $m = 9.11 \times 10^{-31}$ kg and the charge is -1.60×10^{-19} C.
(d) Extra Credit.(5 points) What is the length L of the plates?

(1)

(a)

$$F_{net\,x} = \frac{kq^2}{a^2} + \frac{kq^2}{(\sqrt{2}a)^2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{kq^2}{a^2} + \frac{kq^2}{4a^2}\sqrt{2}$$

$$= \frac{kq^2}{a^2}\left(1 + \frac{\sqrt{2}}{4}\right)$$

(b)

$$F_{net\,y} = 0 + \left(-\frac{kq^2}{(\sqrt{2}a)^2}\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{kq^2}{4a^2}\cdot\sqrt{2}$$

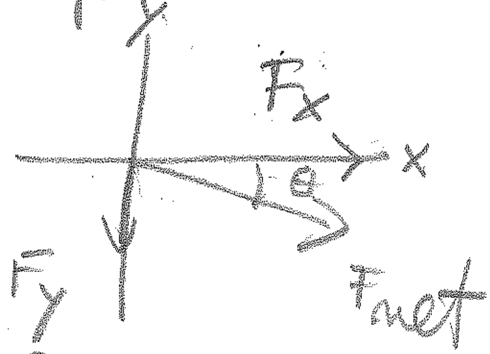
(c)

$$\tan\theta = \frac{\left|-\frac{kq^2}{4a^2}\sqrt{2}\right|}{\left|\frac{kq^2}{a^2}\left(1+\frac{\sqrt{2}}{4}\right)\right|}$$

$$= \frac{\sqrt{2}/4}{1+\sqrt{2}/4} = \frac{0.3534}{1.3534}$$

$$\tan\theta = 0.2611$$

$$\theta_r = 14.633^\circ$$



(d)

$$F_{net} = \frac{kq^2}{a^2}\cdot\sqrt{\left(1+\frac{\sqrt{2}}{4}\right)^2 + \left(-\frac{\sqrt{2}}{4}\right)^2}$$

$$\left(1+\frac{\sqrt{2}}{4}\right) = 1.3534;\ \frac{\sqrt{2}}{4} = 0.3534$$

$$F_{net} = \frac{kq^2}{a^2}\cdot\sqrt{(1.3534)^2 + (0.3534)^2} = 1.40\cdot\frac{kq^2}{a^2}$$

(e)

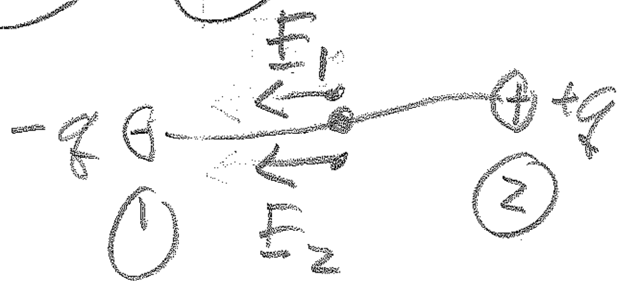
$$F_{net} = \frac{9\times10^9\,(2\times10^{-6})^2}{(1)^2}\sqrt{(1.3534)^2 + (0.3534)^2}$$

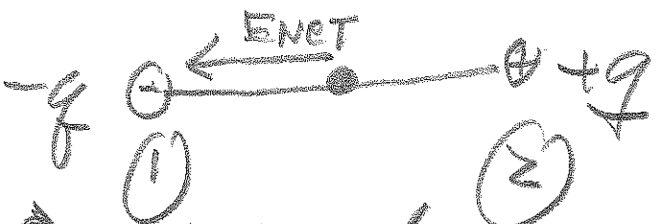
$$= 36\times10^{-3}\cdot\sqrt{1.95658}$$

$$= (36\times10^{-3})(1.3988)$$

$$= 50.3\times10^{-3} = 0.0503\,(N)$$

(2.) (a)





$\vec{E}$ points from +ve, TOWARD -ve.

(b.)

$$E_1 = E_2 = \frac{kq}{d^2}$$

$$E_{net} = \frac{2kq^2}{d^2}$$

$$E_{net} = \frac{2(9\times10^{9})(1\times10^{-19})}{(0.300\times10^{-12})^2}$$

$$= \frac{18\times10^{-10}}{0.09\times10^{-24}}$$

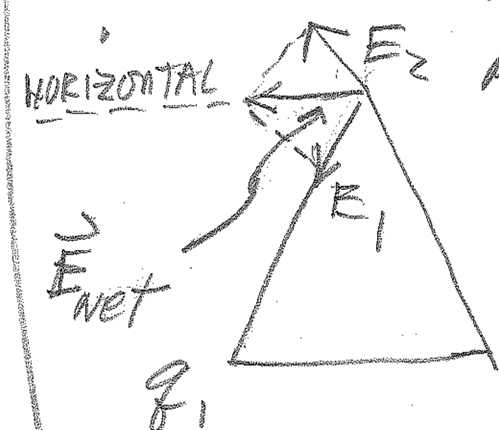
$$= 200\times10^{14}\ N/C$$

$$= 2.00\times10^{16}\ N/C$$

(c) $\cos\theta = \frac{0.300}{0.600} = \frac{1}{2}$

$\rightarrow \theta = 60^\circ$

(d.) 0 from symmetry and parallelogram rule.



(e.) $E_{net} = -2E\cos\theta$

where $E = E_1 = E_2$.

$$E = \frac{kq}{D^2} = \frac{(9\times10^{9})(1\times10^{-19})}{(0.6\times10^{-12})^2}$$

(e.)

$$E = \frac{9\times10^{-10}}{0.36\times10^{-24}}$$

$$= -25\times10^{14}$$

$$= -2.5\times10^{15}\ \frac{N}{C}$$

$$= -0.25\times10^{16}\ \frac{N}{C}$$

$$E_{net_x} = -2(2.5\times10^{15})\cos 60$$

$$= -2.5\times10^{15}\ \frac{N}{C}$$

(f.) ← see PART d.

(g.)

$$E_{net} = |E_{net_x}|$$

$$= +2.5\times10^{15}\ \frac{N}{C}$$

(3.) (a.)

$$D = \sqrt{d^2 + h^2}$$

$$D = \sqrt{(0.05)^2 + (0.12)^2}$$

$$= \sqrt{0.0025 + 0.0144}$$

$$= \sqrt{0.0169}$$

$$= \sqrt{169\times10^{-4}} = 13\times10^{-2}$$

$$= 0.13\ (m)$$

$$= 13\,cm$$

(b.)

$$V = \frac{2ke}{0.130\ (m)}$$

$$= \frac{2(9\times10^{9})(1.6\times10^{-19})}{0.130}\ (V)$$

$$221.54\times10^{-10} = 2.22\times10^{-8}\ (V)$$

(c)

$$KE_i + U_i = KE_f + U_f$$

(3)

(C)

$$0 + e \cdot V_i = KE_f + 0$$

$$\frac{e \cdot 2Ke}{D} = \frac{1}{2} m v_f^2$$

$$v_f = \sqrt{\frac{9.11\times10^{-31}}{1.67\times10^{-27}}} \cdot 88 \frac{m}{s}$$

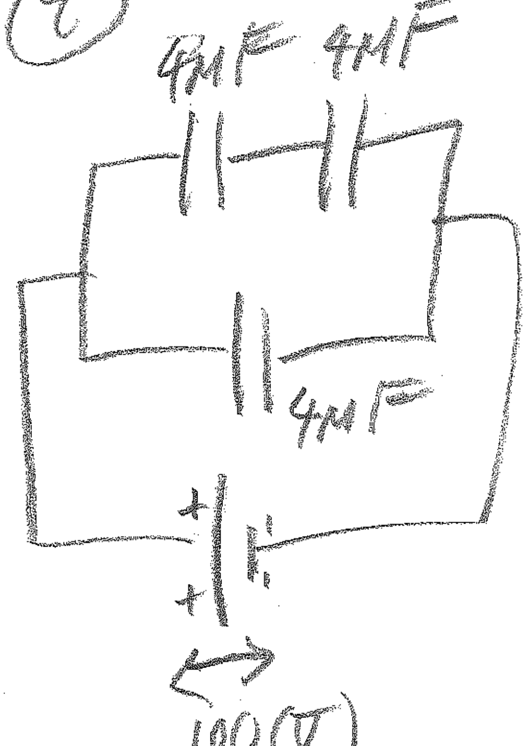
$$= \sqrt{\frac{0.911\times10^{-30}}{1670\times10^{-30}}} \cdot 88 \frac{m}{s}$$

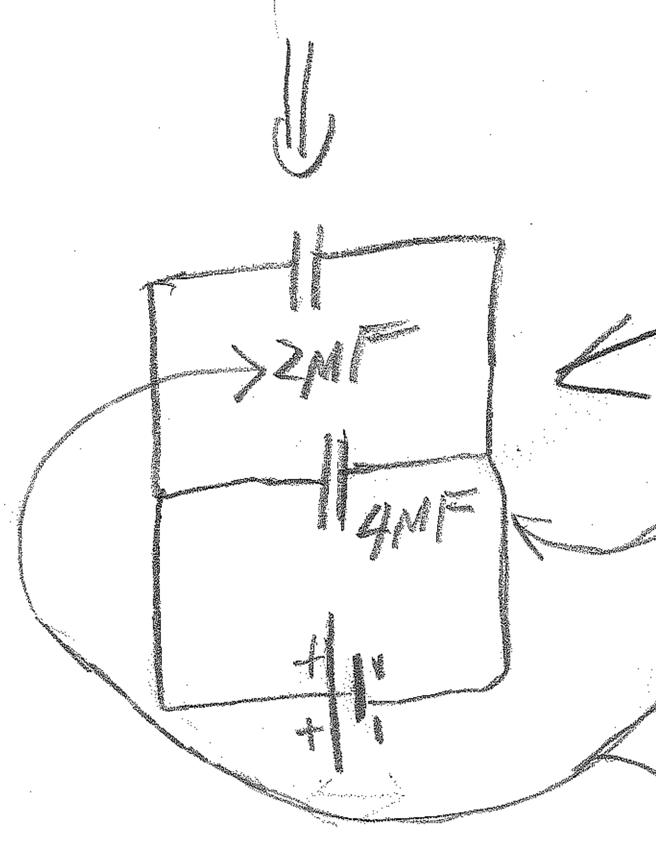
$$= 0.0233 \times 88 \frac{m}{s}$$

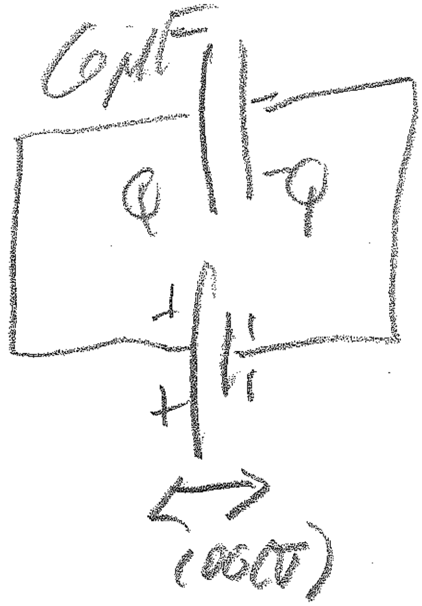
$$= \boxed{2.1 \frac{m}{s}}$$

8.

(a)







$C_{eq} = 6\mu F$

(b) $Q = C_{eq} \cdot \Delta V$

$Q = (6\times10^{-6})(1\times10^{2})(C)$

$Q = 6\times10^{-4} = 600\mu C$

(c.) This is easy if you use reasoning:

$4\mu F: Q_4 = (4\times10^{-6})(10^{2}) = 400\mu C$

$2\mu F: Q_2 = (2\times10^{-6})(10^{2}) = 200\mu F$

<u>200μC</u> across original 4μFs.

(5.)

(a.) TOP;

attraction of negative electron to positive plate.

(b.) down; $\vec{E}$ points from positive to negative.

(c.) $v_y = -\frac{eE_y}{m} \cdot t$

$v_y = a_y \cdot t$

$v_y = \frac{-(1.6\times10^{-19})(-12.00)}{9.11\times10^{-31}}(0.500\times10^{-6})$

$= 1.05\times10^{6}\ \frac{m}{s}$

(d.)

$L = v_x \cdot t$

$L = (6.00\times10^{6})(0.500\times10^{-6})$

$= 3\ m$