test 3 solutions 2A Fall '13

1. (a.) ↓

(b.) negative: motion opposite force in direction

(c.) negative: force opposite motion in direction.

(d.) WORK method:

WORK = ΔKE

$W_g + W_{f_K} = \Delta KE$

$-mgH - 4J = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$$H = \frac{\frac{1}{2}m(v_2^2 - v_1^2) + 4J}{-mg}$$

$$H = \frac{\frac{1}{2}(0.145)(2^2 - 16^2) + 4}{-(0.145 \cdot 9.8)}$$

$$H = \frac{\frac{1}{2}(0.145)(4 - 256) + 4}{-1.421}$$

$$H = \frac{0.29 - 18.56 + 4}{1.421}$$

$H = 10.04\ (m)$

OR energy method:

$KE_1 + U_1 = KE_2 + PE_2 + \text{heat}$

$\frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_2^2 + mgH + 4J$

$\frac{1}{2}(0.145)(16)^2 = \frac{1}{2}(0.145)(2)^2 + (0.145)(9.8) \cdot H + 4$

$\Rightarrow 18.56 = 0.29 + 1.421 \cdot H + 4$

$$H = \frac{18.56 - 0.29 - 4}{1.421} = \boxed{10.04\ (m)}$$

(c) $f_k = \frac{4\,J}{10.04} = 0.398\ N$

(a) (2.) $\frac{1}{2}mv_1^2 = ?$

$KE_1 + U_1 = KE_2 + U_2$

$\frac{1}{2}mv_1^2 + mgh = 0 + 0 + \frac{1}{2}kh^2$

$\frac{1}{2}mv_1^2 = \frac{1}{2}kh^2 - mgh$

$KE = \frac{1}{2}(60)(0.5)^2 - (1)(9.8)(0.5)$

$= 30(0.25) - 4.9$

$= 7.5 - 4.9$

$= 2.6$ Joules (J)

ALTERNATIVE METHOD:

$\Delta KE = \text{WORK}$

$0 - \frac{1}{2}mv_1^2 = mgh - \frac{1}{2}kh^2$

$-\frac{1}{2}mv_1^2 = mgh - \frac{1}{2}kh^2$

$\frac{1}{2}mv_1^2 = \frac{1}{2}kh^2 - mgh$

↓

$KE = 2.6\ J$

(b) $\frac{1}{2}mv_1^2 = 2.6$

$v_1^2 = \sqrt{\frac{(2)(2.6)}{1}}$

$v_1 = 2.28\ \frac{m}{s}$

(c) $W_s < 0$

$W_s = -\frac{1}{2}kh^2$

$W_s = -\frac{1}{2}(60)(0.25)$

$= -7.5$ Joules (J)

(d) $W_g > 0$

$W_g = +mgH$

$= +(1)(9.8)(0.5)$

$= +4.9$ (J)

(3)

(a) $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$(0.5)(2.950) + 0 = (0.5) v_{1f} + 4 \cdot v_{2f}$

$v_{1i} - v_{2i} = v_{2f} - v_{1f}$

$2.950 \ \frac{m}{s} = v_{2f} - v_{1f}$

$\Rightarrow 1.475 = 0.5 v_{1f} + 4 v_{2f}$ (1)

$2.950 = v_{2f} - v_{1f}$ (2)

SUB (2) INTO (1)

(2): $v_{2f} = 2.950 + v_{1f}$

$\Rightarrow$ (1): $1.475 = 0.5 v_{1f} + 4(2.950 + v_{1f})$

$1.475 = 4.5 v_{1f} + 11.8$

$v_{1f} = \frac{1.475 - 11.8}{4.5}$

$= -2.29 \ \frac{m}{s}$

(b)

$v_{2f} = 2.950 + v_{1f}$

$= 2.950 + (-2.29)$

$= 0.66 \frac{m}{s}$

(a) (b) using formulas

$v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_i$

$= \frac{(0.5 - 4)}{4.5} (2.950)$

$= -2.29 \frac{m}{s}$

$v_2 = \frac{2m_1}{(m_1 + m_2)} (2.950)$

$= \frac{(2)(0.5)}{4.5} (2.950)$

$= 0.66 \ m/s$

(c)

$KE_f = \frac{1}{2}(0.5)(-2.29)^2 + \frac{1}{2}(4)(0.66)^2$

$= 1.311025 + 0.8712$

$= 2.18 \ J$

$KE_i = \frac{1}{2}(0.5)(2.950)^2$

$= 2.18 J$

(d) yes since collision is ELASTIC!

(4) (a) AND (b)

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$

$$v_f = \frac{m_1 v_1 + m_2 v_2}{(m_1 + m_2)}$$

$$v_f = \frac{M(24) + M(-32)}{2M}$$

$$= \frac{1}{2}(24 - 32)$$

$$= -4 \frac{m}{s}$$

↑

left!

(c) $\frac{1}{2}(2)(-4)^2$

$= 16 J$

(4.) (d.)

$\frac{1}{2}(1)(24)^2 + \frac{1}{2}(1)(-32)^2$

$= 0.5(24^2 + 32^2)$

$= 800\,J$

(e) heat

$= 800 - 16$

$= \boxed{784\,J}$

(5.)

(a.) $w_2 = w_1 + \alpha \cdot t$

$16700 = 6280 + (1256) \cdot t$

$t = 8.3\ (s)$

(b.) $w_2 = w_1 + \alpha \cdot (10s)$

$w_2 = 6280 + (1256)(10)$

$w_2 = 6280 + 12560$

$= 18840\ \frac{RAD}{s}$

(c.) $w_2^2 = w_1^2 + 2\alpha \cdot \Delta\theta$

$\Delta\theta = \frac{(18840)^2 - (6280)^2}{(2)(1256)} = 125600\ RAD$

$\# \text{REVS} =$

(c) $\dfrac{125600}{2\pi} = 19990$ REVS.

note:

$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$

$\Delta\theta = (6280)(10) + \frac{1}{2}(1256)(10)^2$

$= 125600 \text{ RAD}$

$\# \text{REVS} = \dfrac{125600}{2\pi}$

$= 19990$ REVS

(6.)

(a.) $w_1 = \dfrac{8.40\ m/s}{0.200\ m}$

$= 42\ RAD/s$

(b.) $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, WHERE $\omega = \dfrac{v}{R}$

$= \frac{1}{2}(1.8)(8.4)^2 + \frac{1}{2}\cdot\frac{2}{5}\cdot MR^2 \cdot \dfrac{v^2}{R^2}$

$= \frac{1}{2}(1.8)(8.4)^2 + \frac{1}{5}MV^2$

$= \frac{1}{2}(1.8)(8.4)^2 + \frac{1}{5}(1.8)(8.4)^2$

$= \frac{7}{10}(1.8)(8.4)^2$

$= \boxed{88.91\ J}$

(6)

(C)

$KE_1 + U_1 = KE_2 + U_2$

$88.91J + 0 = 0 + MgH$

$$H = \frac{88.91J}{(1.8)(9.8)}$$

$= 5.04 (m)$

(7.) **SUMMARY:**

$M = 3600\,kg$

$R = 4.0\,cm$

$\tau = 2 \cdot R \cdot F$ and $I \cdot \alpha = \tau$

(a.) $I = \frac{1}{2} M R^2$

$= (0.5)(3600)(4)^2$

$= 28800\ kg\text{-}m^2$

← units!

(b.) $(2)(4)(25) = 200\ N \cdot m$

(c.) $\alpha = \frac{\tau}{I} = \frac{200\ N \cdot m}{28800\ kg\text{-}m^2}$

$= 6.944 \times 10^{-3}\ \frac{RAD}{S^2}$

(d.) $\omega = \alpha \cdot t$

$= (6.944 \times 10^{-3})(60) = \boxed{0.416\ \frac{RAD}{S}}$

(8.)

$ma = mg - T$

and

$I\alpha = R \cdot T = \tau$

$ma = mg - T$ (I)

$\alpha = a/R$

$I \frac{a}{R^2} = T$ (II)

we want $\tau = R \cdot T \Rightarrow$ need a and T

ADD (I) + (II)

$a \cdot (I/R^2 + m) = mg$

$a = \frac{mg}{\left(\frac{I}{R^2} + m\right)} = \frac{(4)(9.8)}{\left(\frac{5}{2} + 4\right)}$

$a = \frac{49}{6.5} = \boxed{6.03 \, m/s^2}$ (b)

(8)

(a) $\tau = R \cdot T$

$\tau = I \cdot \alpha$

$\tau = \frac{1}{2}MR^2 \cdot \frac{a}{R} = \frac{1}{2}MR \cdot a$

$\tau = \frac{1}{2}(5) \cdot (0.150)(6.03)$

$= 2.26\ N \cdot m$

(c) $v^2 = 2ah$

$v = \sqrt{2ah}$

$= \sqrt{2(6.03)(2)} = 4.91\ \frac{m}{s}$

(a) **ENERGY:** (CH.9)

$$mgH = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$\omega = \frac{v}{R}$$

$$mgH = \frac{1}{2}\cdot\frac{1}{2}MR^2\frac{v^2}{R^2} + \frac{1}{2}mv^2$$

$$mgH = \frac{1}{4}Mv^2 + \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{m\cdot g\cdot H}{\frac{1}{4}M + \frac{1}{2}m}}$$

$$v = \sqrt{\frac{(1)(9.8)(1)}{0.5 + 0.5}} = 3.13\,\frac{m}{s}$$

(9.) DYNAMICS (CONT.)

$$ma = mg - T \quad (i)$$

$$I\frac{a}{R} = R \cdot T \Longleftrightarrow I\alpha = \tau$$

$$I\frac{a}{R^2} = T \quad (ii)$$

ADD (i.) + (ii.)

$$a\left(m + \frac{I}{R^2}\right) = mg$$

$$a = \frac{mg}{m + \frac{I}{R^2}} = \frac{mg}{m + \frac{M}{2}}$$

$$a = \frac{(1)(9.8)}{1+1} = 4.9\,\frac{m}{s^2}$$

(9) dynamics

$v = \sqrt{2 \cdot a \cdot h}$ from CH 2.

$v = \sqrt{(2)(4.9)(1)}$

$v = 3.13 \, m/s$