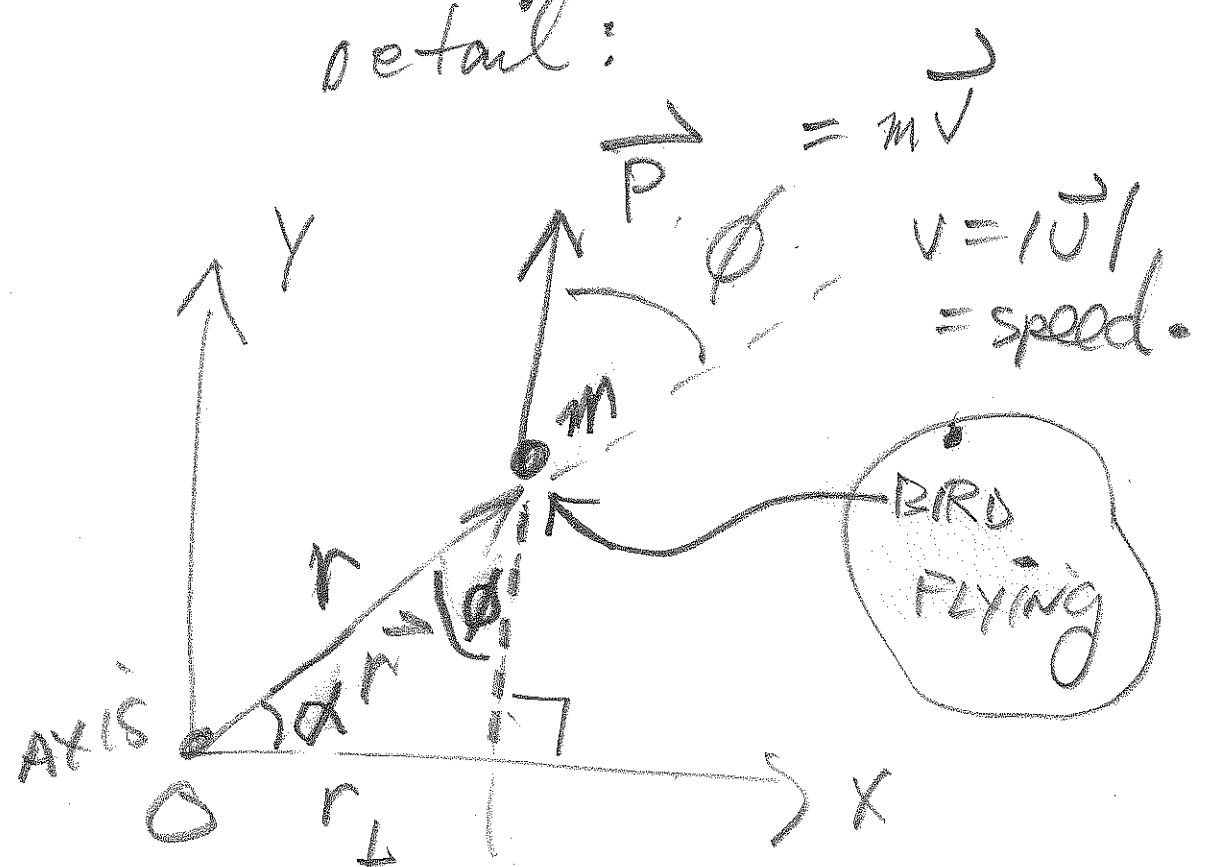


2A 10-31-13
CNIO

angular momentum in

detail:

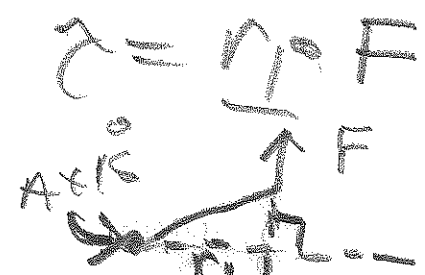


$$|\vec{L}| = r_{\perp} \cdot p$$

$$|\vec{L}| = \boxed{r \cdot \cos \alpha \cdot m v}$$

$\alpha = 90 - \phi$

line of action
 LIFE TORQUE:

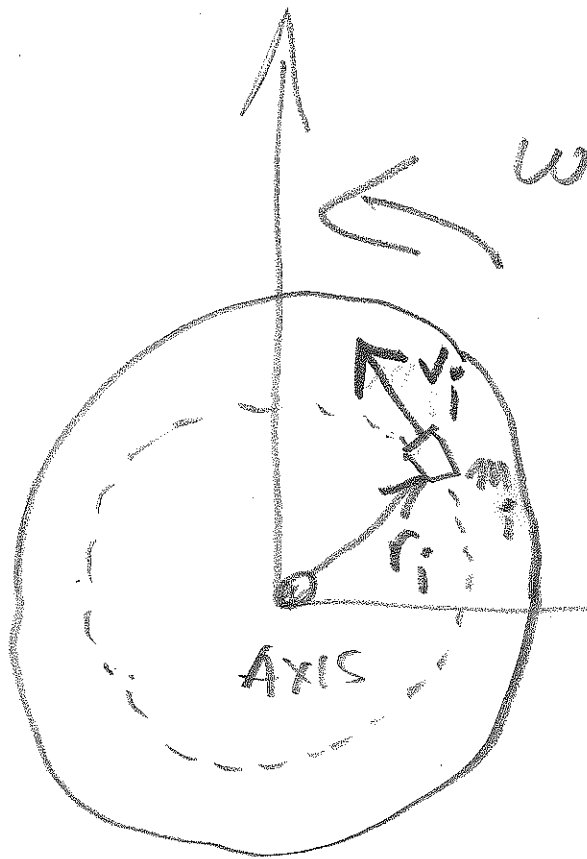


$$\tau = r_{\perp} \cdot F$$

note: $|\vec{L}| = r \sin \phi \cdot m v$
 $\cos \alpha = \sin \phi = \cos(90 - \phi)$

THEORETICAL TRICK:

pretend you have a SOLID
 CYLINDER (SOLID). SHOWN
 is the i^{th} MOLECULE.



$$L_i = r_i \sin 90 \cdot m_i v_i$$

$$= r_i m_i v_i$$

$$= r_i m_i r_i \omega$$

$$= r_i^2 m_i \omega$$

$$= m_i r_i^2 \omega$$

$$= m_i r_i^2 \omega$$

$$N = 6 \times 10^{25}$$

$$L_{\text{TOTAL}} = \sum_{i=1}^N m_i r_i^2 \omega = I \omega$$

$$I = \sum_{i=1}^N m_i r_i^2$$

Conservation of angular momentum

video: WORLD

RECORD ice skate

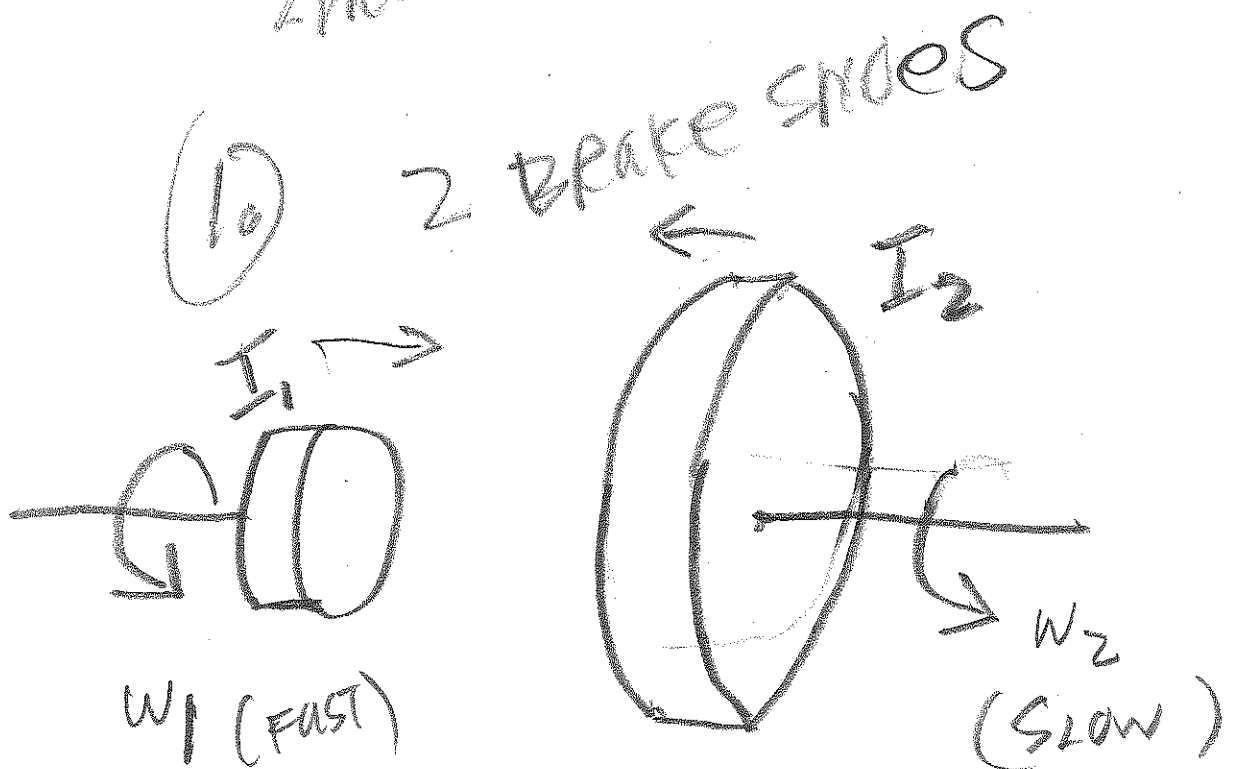
SPINNING!

Natalia

Kanounnikova

~ $\frac{5 \text{ rev}}{\text{second}}$!

2 classical
examples of
conservation of
of angular
momentum:

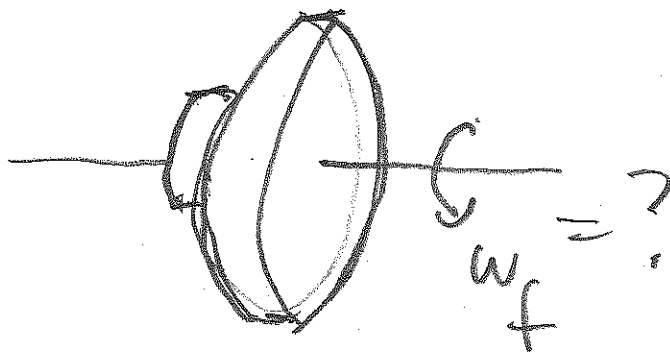


(A) 2 BRAKES come into contact and stick together.

what is final

common angular speed?

contact (stick); TURN TOGETHER.

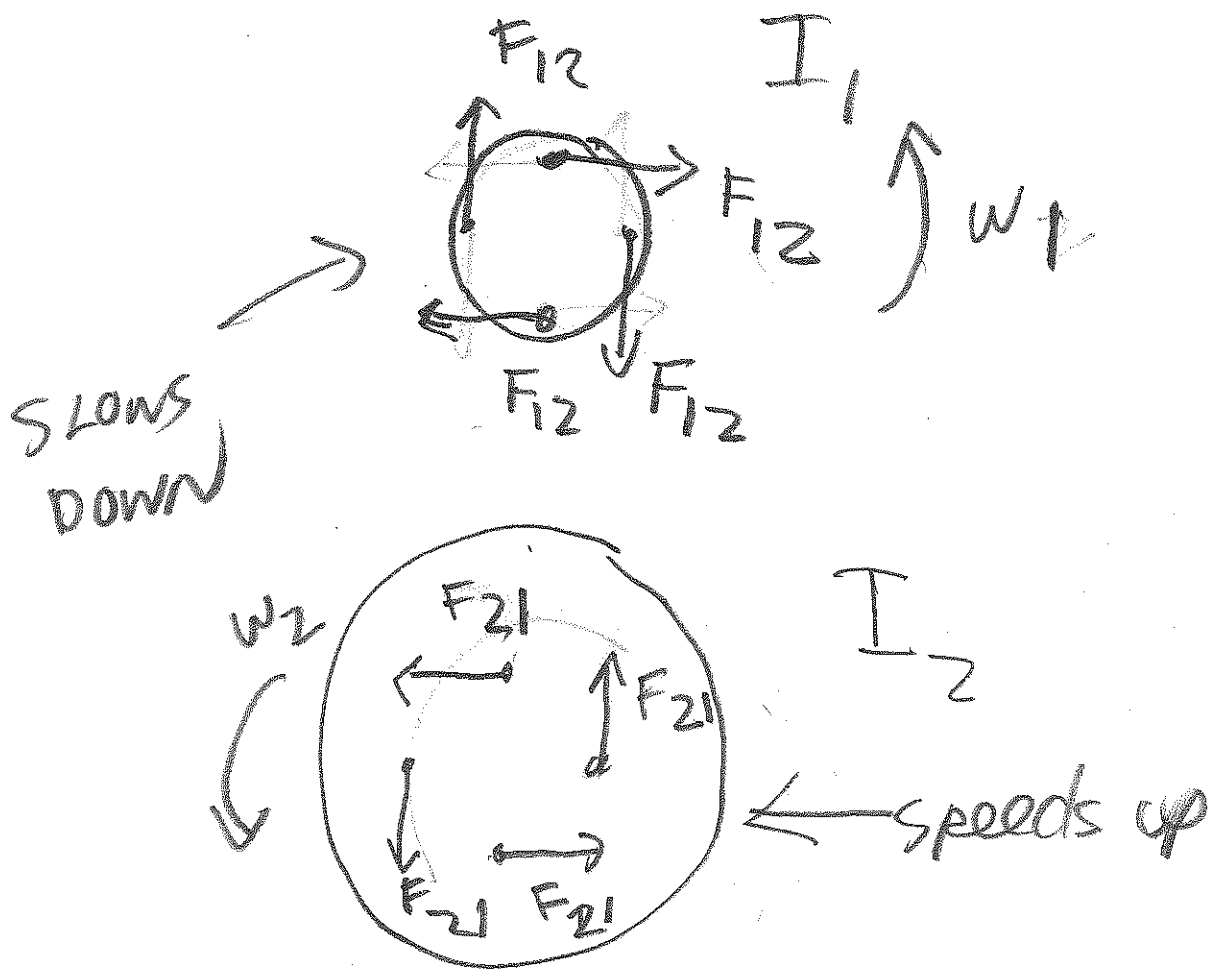


conservation of angular momentum

$$L_i = L_f$$

$$I_1 \omega_1 + I_2 \omega_2 = I_1 \omega_f + I_2 \omega_f$$

(1) NOTE: when they first come into contact:



NS 3RD LAW: $F_{21} = F_{12}$
 Equal and opposite torques
 net torque = 0 \Rightarrow

$$\Delta L_1 + \Delta L_2 = 0$$

$$\frac{\Delta L_1}{\Delta t} + \frac{\Delta L_2}{\Delta t} = 0$$

$$\tau_{12} + \tau_{21} = 0$$

$$\tau_{12} = -\tau_{21}$$

Thus ang. mom. is conserved

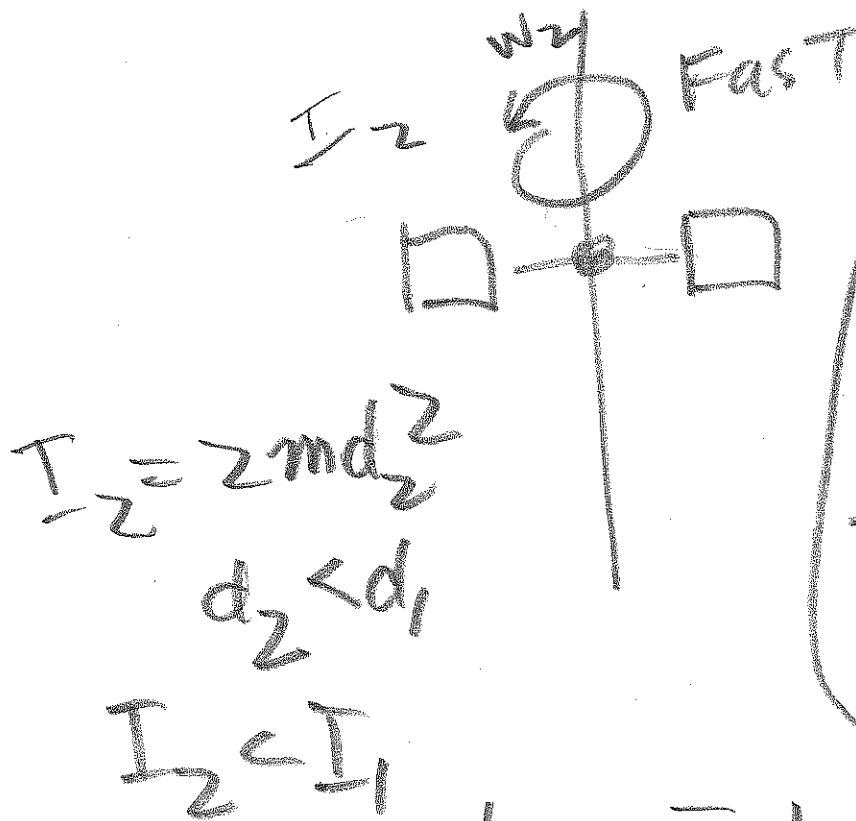
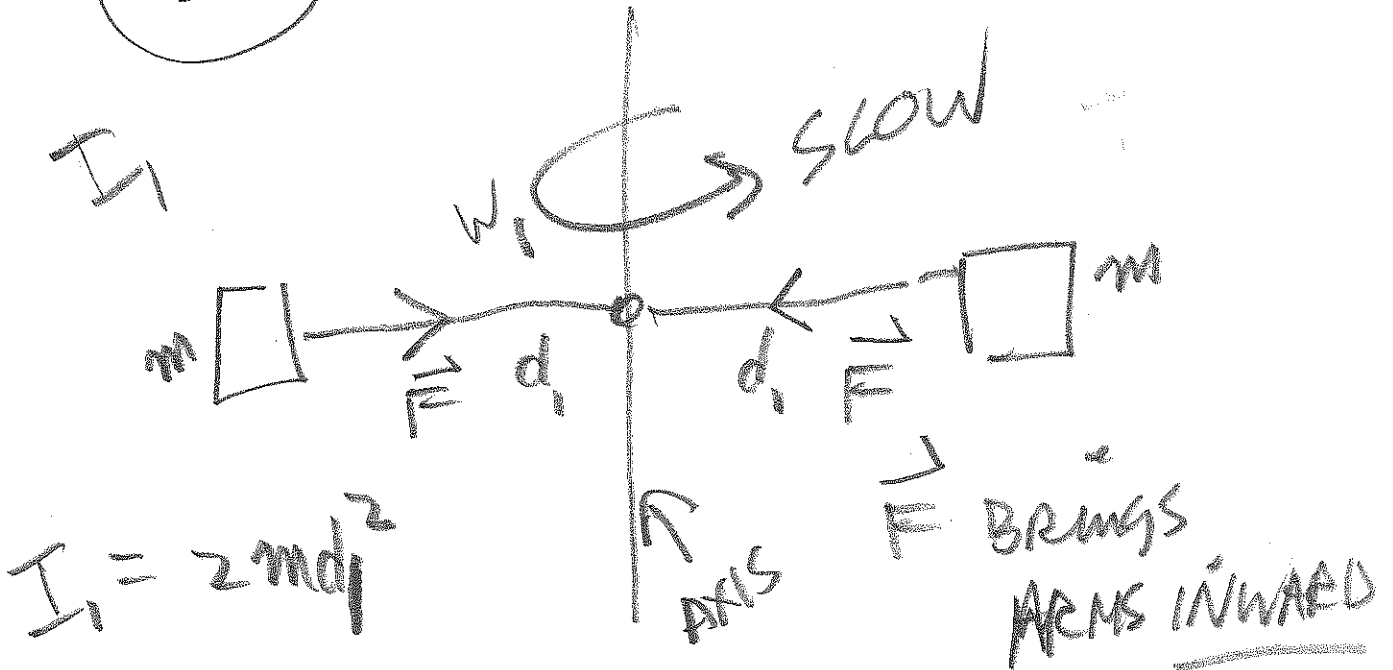
$$I_1 \omega_1 + I_2 \omega_2 = (I_1 + I_2) \omega_f$$

$$\omega_f = \frac{I_1 \omega_1 + I_2 \omega_2}{(I_1 + I_2)}$$

$$\boxed{\omega_2 < \omega_f < \omega_1}$$

(2)

ice skater pulls in arms



BUT F POINTS THROUGH AXIS OF ROTATION.
 THUS F CAUSES NO TORQUE.

$L_i = L_f$

$I_1 w_1 = I_2 w_2 \Rightarrow w_2 = \frac{I_1}{I_2} w_1$

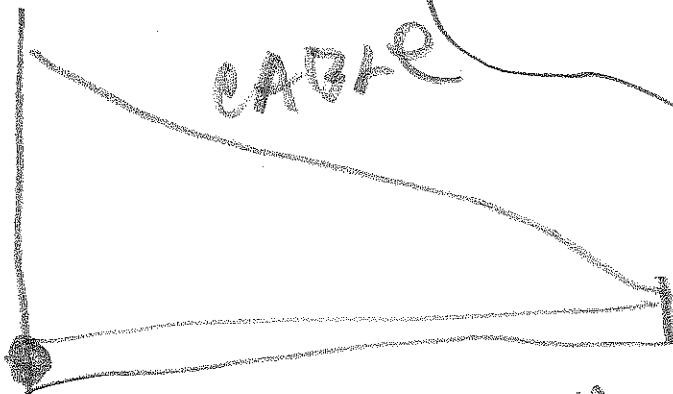
NOTE $w_2 > w_1$

STATICS 10.6

2 classical problems

I.

WALL



CONSTRUCTION BEAM HELD UP BY A CABLE

AXIS OF ROTATION

BEAM

II.

LADDER

EXAMPLE 10.12

AXIS OF ROTATION

PERSON MOVES UP

WALL

NO FRICTION

* AXIS MOVES LEFT IF LADDER ~~SLIPS~~ SLIPS.

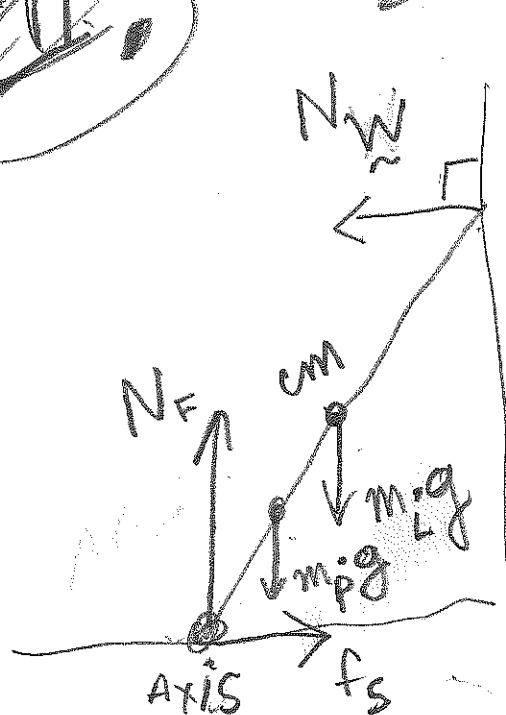
FLOOR

DANGER!

mastering pre-lecture problem

(II)

EXAMPLE 16.12



m_p = MASS
of person
on ladder

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum \tau = \curvearrowleft - \curvearrowright = 0$$

AXIS

(a) FIND N_W , N_F , f_s

(b) FIND M_s IF $f_s \approx f_{sMAX}$

(c) FIND vector sum of \vec{f}_s and N_F

LADDER EXAMPLE 16.12

(II.)

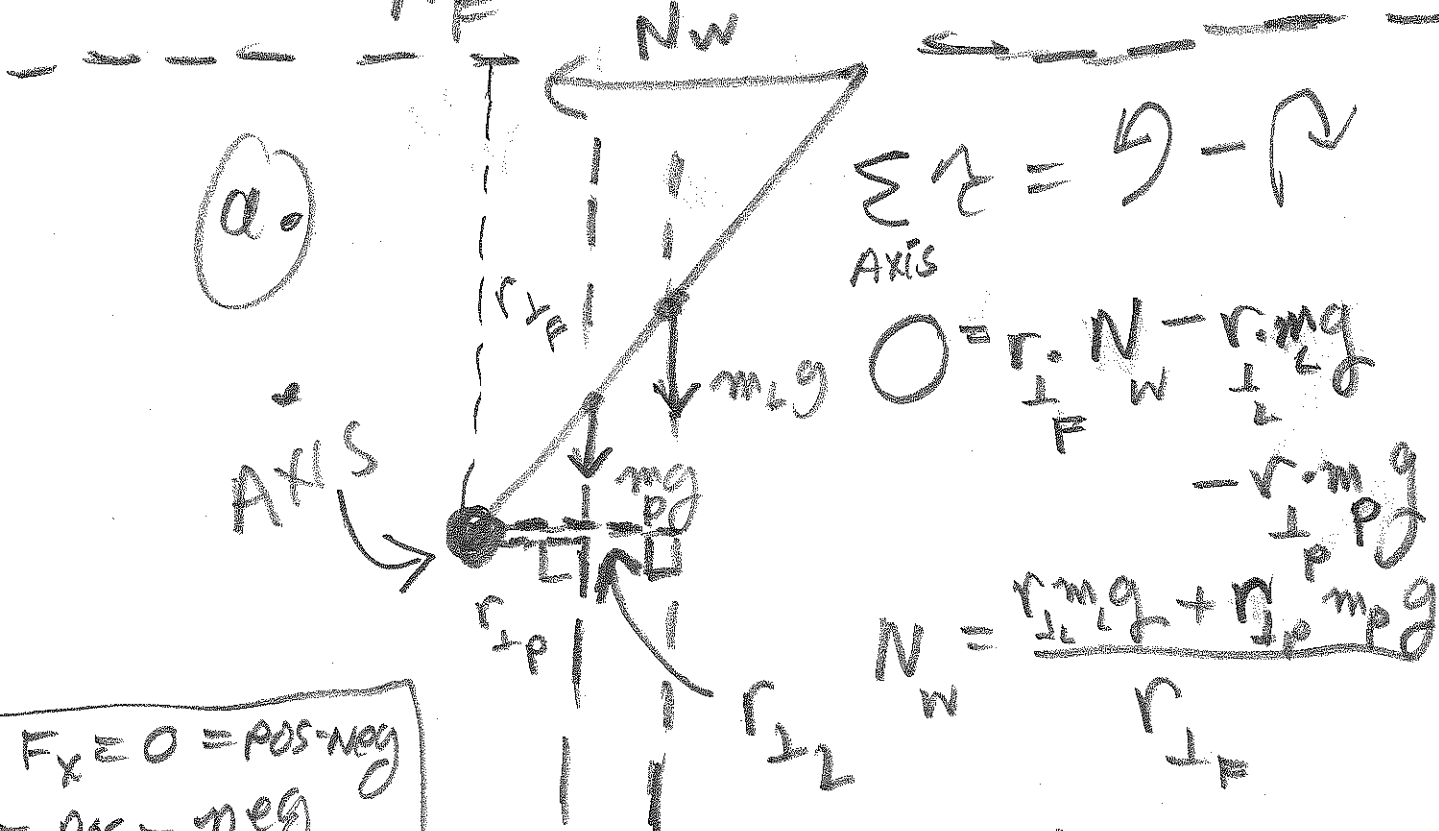
(a)

$$\sum F_y = 0 = N_F - m_p g - m_L g$$

$$N_F = m_p g + m_L g$$

$$N_F = 800 \text{ (N)} + 180 \text{ (N)}$$

$$N_F = 980 \text{ (N)}$$



(a)

$$\sum \tau = 0 - \tau$$

AXIS

$$0 = r_{\perp F} \cdot N - r_{\perp L} \cdot m_L g - r_{\perp P} \cdot m_p g$$

$$N = \frac{r_{\perp L} m_L g + r_{\perp P} m_p g}{r_{\perp F}}$$

$$= 2080 \text{ (N)}$$

$$\sum F_x = 0 = \text{pos} - \text{neg}$$

$$0 = \text{pos} - \text{neg}$$

$$0 = f_s - N_W$$

$$f_s = N_W = 2080 \text{ (N)}$$