

11.14

SEE DATA SHEET FOR THIS LAB.

In this experiment, you measure the parameters --mass m and spring constant k -- of an oscillating system and check whether the period T is as theory predicts. Consider the mechanical oscillator illustrated in figures 12-7 in textbook and my notes. At equilibrium, the mass has zero force on it; there the spring is neither stretched or compressed. When x is negative, the spring is compressed. Otherwise it is stretched. $F_x = -kx$ is the x -component force causing the mass to return to the origin ($x = 0$) at all times; when the mass moves away from the origin, it slows down and when it moves toward the origin it speeds up.

This explains all kinds of oscillators, whether they be molecules vibrating back and forth in the warm pen you are using to write your lecture notes, air molecules oscillating back and forth as my voice travels across the lecture room, or an "up and down" moving piece of string supporting a transverse standing wave such as on a guitar string. Such oscillations are everywhere, including your quartz watch.

In Chapter 19, see discussion of VIBRATIONS OF A PENDULUM. See page 335. The analogous formula for a mass m on a spring of force constant k is :

$$T_{TH} = 2\pi \sqrt{\frac{m}{k}} \quad (1)$$

← Theoretical T

Note that you may choose the positive x direction to be downward which means x is positive when the vertical spring is stretched. To test the theory, you will measure the parameters (Mass m , Spring Constant k) that appear in the theoretical expression for T to obtain a theoretical value T_{TH} . --see equation 1. By a timing procedure, you will directly measure an experimental value T_{EX} , and then compare T_{TH} with T_{EX} . (See attached data sheet.) In particular, you do the following for the mass-spring system:

- Measure the parameters m and k that appear in your expression for the period in equation (1) and use them to compute T_{TH} .
- Measure the time t for a known number of cycles n ($n = 10$) and compute the experimental value of the period $T_{EX} = t/n$. You will measure T_{EX} 5 times and compute the average T_{EX_BEST} .
- Compute the PERCENT ERROR between T_{TH} AND T_{EX} , using the formula:

$$\frac{|T_{EX} - T_{TH}|}{\text{AVERAGE}}, \text{ WHERE AVERAGE IS THE AVERAGE OF } T_{EX} \text{ AND } T_{TH}.$$

PROCEDURE

As in figure 12-7, you will hang a tapered spring vertically from a support with the *small end* of the spring at the *top*. This spring taper direction is important for an accurate reading. The narrowing of the spring's diameter toward the top allows the spring to stretch uniformly along its entire length. The narrowing toward the top means the upper half of the spring is stiffer than the lower half. This stiffness "gradient" is necessary because the upper part supports the weight of the lower part and so must be stiffer if it is to have the same stretch per unit length as the lower.

(Q-1) You will add a mass of value $m = 0.30$ kg as suggested in figure 12-7 and allow the system to oscillate up and down. Remember, the hanger already has mass 50 g, so you will add 0.250 kg. The formula for the period is given by equation (1) but there is one complication that has to do with the correct value of the mass. While the system vibrates up and down, different parts of the spring move at different speeds. The effective mass $m_{s,eff}$ of the spring adds to the system's inertia and kinetic energy storage capacity. It turns out that the mass m includes both 0.30 kg *and* one-third of the mass of the spring; a calculus derivation shows that $m_{s,eff} = m_s/3$, where m_s is the spring mass. You may enter this expression under (Q-1) of the attached data sheet.

(Q-2) To find the value of the spring constant k , use a sliding caliper jaw to record the position x' of the bottom of the weight hanger with weights of mass $M = 100, 200, 300, 400$ and 500 grams (g) suspended from the spring. Note: The uncertainties in x' and the hanging masses should be very small. Thus, the precision of your value of k will be limited by the COMPUTER PROCESS. WE WILL USE MICROSOFT EXCEL OR VERNIER'S LOGGER PRO.

(Q-3) Weigh the spring to find m_s and compute the effective mass $m_{s,eff}$. Enter the value $m_0 = 0.30$ kg. Compute the total mass $m = m_{s,eff} + m_0$. Use this value

along with k to find $T_{TH} = 2\pi \sqrt{\frac{m}{k}}$

(Q-4) Suspend 0.30 kg from the spring, displace it slightly and determine the experimental period T_{EX} .

Make at least 5 trials and count at least 10 cycles for each trial:

You will measure T_{EX} 5 times and compute the average $T_{EX-BEST}$. From this, you will also compute R/N , where $N = 2$ and $R = (\max - \min)$ just like picket fence lab; this gives the statistical uncertainty of multiple measurements.

(Q-5) Compare the discrepancy between the theoretical and experimental values with the sum the uncertainty R/N . ALSO Compute the PERCENT ERROR between T_{TH} AND T_{EX} , using the formula:

$\frac{|T_{EX} - T_{TH}|}{\text{AVERAGE}}$, WHERE AVERAGE IS THE AVERAGE OF T_{EX} AND T_{TH} .

(Q-1)
 RECORD $M_{EFF} = m_s/3$, where m_s is the spring mass.
 $\frac{m_s}{3} \approx 53g = 0.053 kg$

(Q-2) FILL IN THE TABLE

M(kg)	F _s (N), using g = 9.8 m/s ²	x' (m)
0.100	0.98	set 0
0.200	1.96	x ₂
0.300	2.94	x ₃
0.400	3.92	x ₄
0.500	4.90	x ₅

to meters
 ÷ 100

Computation of k; WE WILL USE MICROSOFT EXCEL OR VERNIER'S LOGGER PRO.

(Q-3)

masses related to total mass m	
m _s	m _s = 160g = 0.160 kg
m _{sEFF}	m _s /3 ≈ 53 g
m ₀	300g = 0.300 kg
m	m ₀ + m _{sEFF} ≈ 353g

0.353 kg

We are now in the position to find the theoretical period given by formula (1):

$$T_{TH} = 2\pi \sqrt{\frac{m}{k}}$$

REVIEW TODAY

(Q-4)

n	time for n cycles (s)	T _{EX} = t/n (s)
10		1.15
10		1.16
10		1.09
10		1.10
10		1.14
		T _{EX BEST} = Average $\frac{T_1 + T_2 + T_3 + T_4 + T_5}{5}$
		R/N = $\frac{\text{MAX} - \text{MIN}}{2}$

NOTE: ABOVE, compute R/N, where N = 2 and R = (max - min) just like picket fence lab.

(Q-5)

Now check the discrepancy, $|T_{EX} - T_{TH}| < R/N$. IS THIS TRUE?

$$|T_{EX} - T_{TH}| < \frac{\text{MAX} - \text{MIN}}{2}$$

ALSO FIND THE PERCENT ERROR = $\frac{|T_{EX} - T_{TH}|}{\text{AVERAGE}}$, WHERE AVERAGE IS THE AVERAGE OF T_{EX} AND T_{TH}

$$P.E. = \frac{|T_{EX} - T_{TH}|}{\left(\frac{T_{EX} + T_{TH}}{2}\right)} \times 100\%$$